Laplace transform

1) Which of the following is the Laplace transform of:

$$f(t) = \begin{cases} 1, & \text{if } \quad 0 \leq t < 2, \\ t^2 - 4t + 4, & \text{if } \quad t \geq 2 \end{cases} \quad ?$$

(a) $F(s) = \dfrac{2e^{-2s}}{s^3}$

(b) $F(s) = \dfrac{1 - e^{-2s}}{s} + \dfrac{2e^{-2s}}{s^3}$

(c) $F(s) = \dfrac{e^{-2s}}{s} + \dfrac{2 - 2e^{-2s}}{s^3}$

(d) $F(s) = \dfrac{2 - 2e^{-2s}}{s^3}$

2) The inverse Laplace of

$$F(s) = \frac{2s + 3}{s^2 + 4s + 13}$$

is

(a) $f(t) = e^{2t}\left(2\cos 3t - \frac{1}{3}\sin 3t\right)$

(b) $f(t) = e^{-2t}\left(2\cos 3t - \frac{1}{3}\sin 3t\right)$

(c) $f(t) = 2\cos 3(t+2) - \frac{1}{3}\sin 3(t+2)$

(d) $f(t) = 2\cos 3(t-2) - \frac{1}{3}\sin 3(t-2)$

3) Suppose that the function y(t) satisfies the differential equation y``- 2y` - y = 1 with initial condition y`(0) = -1, y(0) = 1, then Laplace transform is

(a) $Y(s) = \dfrac{1}{s^2 - 2s - 1}$

(b) $Y(s) = \dfrac{1}{s(s^2 - 2s - 1)}$

(c) $Y(s) = \dfrac{s+1}{(s^2 - 2s - 1)} + \dfrac{1}{s(s^2 - 2s - 1)}$

(d) $Y(s) = \dfrac{-s+3}{(s^2 - 2s - 1)} + \dfrac{1}{s(s^2 - 2s - 1)}$

4) By using definition, find Laplace transform of e^{at}, t^2, cosat

5)) $\int_0^\infty t \cos 3t\, e^{-5t}\, dt$ = (-4/289, 4/289, 15/578, -15/578)

6) $L\{t^2\, U(t-4)\} = \left(\dfrac{2}{s^3}e^{-4s},\quad \int_0^\infty t^2\, e^{-st}\, dt,\quad \dfrac{2}{s^3}(8s^2+4s+1)e^{-4s},\quad \int_4^\infty t^2\, e^{-st}\, dt\right)$

7) $L^{-1}\left\{\dfrac{1}{s^2(s+9)}\right\} = \left(\dfrac{e^{9t}-1}{9},\quad \dfrac{1-e^{-9t}}{9},\quad \int_{u=0}^{t} \dfrac{1-e^{-9u}}{9}\, du,\quad \int_{u=0}^{t} \dfrac{e^{9u}-1}{9}\, du\right)$

8) $L\{\sin 3t\, U(t-2)\} = \left(\dfrac{3}{s^2+9}e^{-2s},\quad \dfrac{1}{s^2+9}((\cos 6)s + 3\sin 6)\, e^{-2s},\right.$

$\left.\dfrac{1}{s^2+9}((\sin 6)s + 3\cos 6)\, e^{-2s},\quad \int_2^\infty \sin 3t\, e^{-st}\, dt\right)$

9) $\int_0^\infty t \sin 2t\, e^{-4t}\, dt$ = (0.03, 0.25, 0.04, 0.4)

10) $L^{-1}\left\{\dfrac{1}{s^2(s-4)}\right\} = \left(\dfrac{e^{4t}-1}{4},\quad \dfrac{1-e^{-4t}}{4},\quad \int_{u=0}^{t} \dfrac{1-e^{-4u}}{4}\, du,\quad \int_{u=0}^{t} \dfrac{e^{4u}-1}{4}\, du\right)$

11) $L^{-1}\{\frac{1}{s^2(s^2-9)}\} = (\frac{1+\cosh 3t}{9},\ \frac{1-\cosh 3t}{9},\ \frac{3t-\sinh 3t}{27},\ \int_{u=0}^{t}\frac{(\cosh 3u-1)}{9}\,du)$

12) $L\{(e^{3t}\sin 3t)\,U(t-2)\} = (\int_{1}^{\infty}\sin 3t\, e^{-(s-3)t}\,dt,\ \frac{3}{(s-3)^2+9}e^{-2s},$

$[\frac{(\sin 6)(s-3)+3\cos 6}{s^2-6s+18}]e^{-2(s-3)}, [\frac{(\cos 6)(s-3)+3\sin 6}{s^2-6s+18}]e^{-2(s-3)})$

13) $\int_{0}^{\infty}(\frac{e^{2t}-\cos 3t}{t})\,e^{-3t}\,dt = (\mathrm{Ln}(18),\quad \mathrm{Ln}(9),\quad \frac{1}{2}\mathrm{Ln}(18),\quad 2\,\mathrm{Ln}\sqrt{\frac{3}{2}})$

14) $L^{-1}\{\frac{1}{s^2(s-4)}\} = (\frac{e^{4t}-1}{4},\ \frac{1-e^{-4t}}{4},\ \int_{u=0}^{t}\frac{1-e^{-4u}}{4}\,du,\ \int_{u=0}^{t}\frac{e^{4u}-1}{4}\,du)$

15) $L\{\sin 3t\,U(t-2)\} = (\frac{3}{s^2+9}e^{-2s},\ \frac{1}{s^2+9}((\cos 6)s+3\sin 6)\,e^{-2s},$

$\frac{1}{s^2+9}((\sin 6)s+3\cos 6)\,e^{-2s},\ \int_{2}^{\infty}\sin 3t\, e^{-st}\,dt)$

16) $\int_{0}^{\infty}(\frac{\cos 2t-\cos 3t}{t})\,dt = (\mathrm{Ln}(\frac{2}{3}),\quad -\mathrm{Ln}(\frac{2}{3}),\quad 0,\quad 2\,\mathrm{Ln}\sqrt{\frac{3}{2}})$

17) Find F(s) of the following functions:

a) $f(t) = 5t+4\sin 2t-2\cosh 3t$, b) $f(t) = te^{-2t}-4\sin^2 3t$

c) $f(t) = e^{-4t}-5t^4+\sin 3t\cosh 2t$, d) $f(t) = t\sin 2t\cos 3t$

e) $f(t) = \sin t\,U(t-\pi) + \frac{e^{2t}-e^{-3t}}{t}$, g) $f(t) = \left\{\begin{array}{ll} 2 & 0<t\le 2 \\ 3t & 2<t\le 3 \\ t^2 & t>3 \end{array}\right\}$,

h) $f(t) = \sin^2 4t + \cos 2t\cosh 5t + t\,H(t-2)$

I) $f(t) = U(t-3)[e^{7t} + 4t]$

J) $f(t) = U(t-3)[-e^{5t} + 2 + 3t^2]$

k) $f(t) = 5\sin(5t+8)$

L) $f(t) = e^{-2.7t}[\cos(9.2t + 3)] + \frac{ke^{-k^2/4t}}{\sqrt{4\pi t^3}} + \frac{e^{-k^2/4t}}{\sqrt{\pi t}}$

18) Find inverse Laplace of the following functions

a) $F(s) = \frac{s}{s^2+4s+9}$, b) $F(s) = \frac{e^{-2s}}{s+3}$, c) $F(s) = \frac{1}{(s+9)^3}$,

d) $F(s) = \frac{1}{(s-1)^3} - \frac{6}{s^2+9}$, e) $F(s) = \frac{4}{s^2-4s+4} + \frac{1}{4-2s}$,

g) $F(s) = \frac{4}{s(s^2+9)} + \frac{e^{-2s}}{s^2+9}$, h) $F(s) = \frac{25}{s^3(s^2+4s+5)}$

I) $F(s) = \frac{4}{s^3-4s} + \frac{4}{s^3+4s}$

J) $F(s) = \frac{2-3se^{-s}+4e^{-3s}}{s(s+1)} + \frac{5s^2+8s-5}{s^2(s^2+2s+5)}$

k) $F(s) = \frac{s}{(s-1)^3} + \frac{25}{s^2(s+1)^2}$

L) $F(s) = \frac{s^3+9s^2+6s+3}{s^3+5s^2+4s+6}$

M) $F(s) = \frac{s^2+3s+5}{6s^2+5} + \frac{9s+4}{(s+3)^3}$

N) $F(s) = \frac{2-3se^{-s}+4e^{-3s}}{s(s+1)} + \frac{9s+4}{(s-3)^2+6}$

P) $F(s) = e^{-4s}[\frac{2}{s(s+1)} - \frac{5}{s}] + \frac{1}{(s+1)^4}$

19) Solve the following differential equations using Laplace:

a) y\` - 2y = 8 , y(0) = 0

b) y\`\`+ y\` - 2y = 0 , y(0) = 0 , y\`(0) = 3

c) y\`\`- 2y\` - 3y = 4 , y(0) = 1 , y\`(0) = 7

d) y\`\`- 3y\` + 2y = 8$e^{2t}$, y(0) = y\`(0) = 3

e) $y'' + y' - 2y = t$, $y(0) = 0$, $y'(0) = 2$

f) $y'' - 3y' + 2y = 20e^{5t}$, $y(0) = 7$, $y'(0) = 2$

g) $3y' + 4y = \sin 2t$, $y(0) = 1/3$

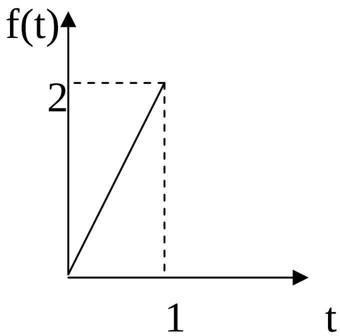
h) $y'' + y = 6\cos 2t$, $y(0) = 3$, $y'(0) = 1$

j) $y'' - y' - 2y = 12e^{-2t}(4\sin t - \cos t)$, $y(0) = 0$, $y'(0) = 3$

k) $y' + y = U(t-1) - U(t-3)$, $y(0) = 2$

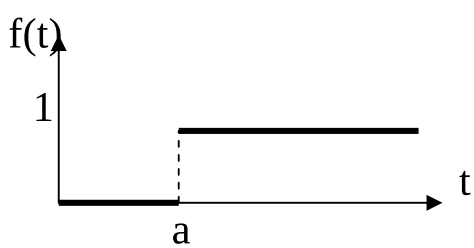
L) $y'' + 2y' - 3y = U(t-2)(t-1)$, $y(0) = 1$, $y'(0) = -1$

M) $y'' + y = f(t)$, $y(\pi/4) = \pi/2$, $y'(\pi/4) = 2 - \sqrt{2}$, where f(t) is given by indicated graph



[Hint: let $y(0) = a$, $y'(0) = b$]

N) $y'' + by = f(t)$, $y(0) = 1$, $y'(0) = 0$, where b is a constant and f(t) is given by indicated graph



20) $Y(s) = \dfrac{1}{(s^2+1)}$ is the Laplace transform derived from:

a) $y'' + y = 0$, $y(0) = 0$, $y'(0) = 0$

b) $y'' + y = 0$, $y(0) = 0$, $y'(0) = 1$

c) $y'' + y = 0$, $y(0) = 1$, $y'(0) = 0$

21) If $Y(s) = \dfrac{s+2}{s^2+2s+2}$, then its inverse Laplace transform is

a) $f(t) = e^{-t}(\sin t + \cos t)$

b) $f(t) = e^{t}(\sin t + \cos t)$

c) $f(t) = e^{t}\cos t$

d) $f(t) = e^{-t}\sin t$

22) If $Y(s) = \dfrac{1- e^{-\pi s}}{s^2+1}$, then its inverse Laplace transform is

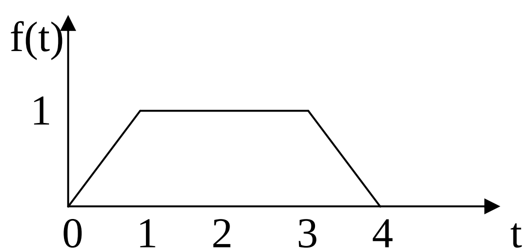
a) $f(t) = \sin t\,[1+ U(t-\pi)]$

b) $f(t) = (1- e^{-\pi t})\sin t$

c) $f(t) = \sin t\,[1- U(t-\pi)]$

d) f(t) is the square wave.

23)



The above graph define the function

a) $L^{-1}\{1- e^{-s} - e^{-3s} + e^{-4s}\}$

b) $L^{-1}[\frac{1}{s}\{1- e^{-s} - e^{-3s} + e^{-4s}\}]$

c) $L^{-1}[\frac{1}{s^2}\{1- e^{-s} - e^{-3s} + e^{-4s}\}]$

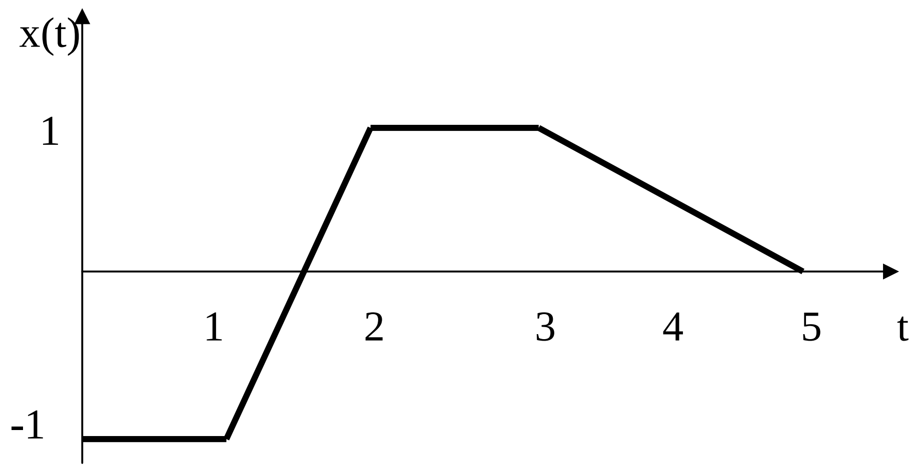
24) Laplace transform of the function $f(t) = \cos t\, U(t-\pi)$ is

$$\left(\frac{e^{-\pi s}}{s^2+1}, \frac{s\,e^{\pi s}}{s^2+1}, -\frac{s\,e^{-\pi s}}{s^2+1}\right)$$

25) Laplace transform of the function $f(t) = \int_0^t \frac{\sin t}{t} dt$ is

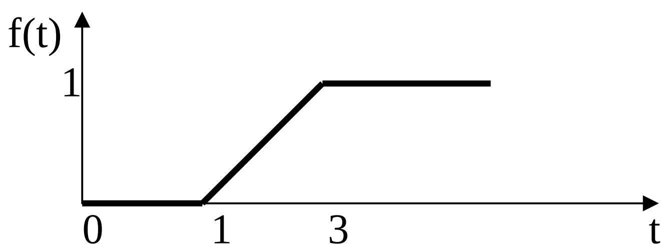
(arctans/s, arccoss/s, arccots/s)

26)



Find Laplace transform of x(t)

27)



i- Write f(t) in terms of unit step function.

ii- Find the Laplace transform of f(t).

28) If $f(t) = \left\{ \begin{array}{ll} 0 & 0 < t < 1 \\ t & t > 1 \end{array} \right\}$, then Laplace transform is

(e^{-s}/s^2 , e^{s}/s^2 , $e^{s}[1/s^2 + 1/s]$, $e^{-s}[1/s^2 + 1/s]$, $e^{-s}[1/s^2 - 1/s]$)

29) If $F(s) = \dfrac{e^{-s}}{s(s+1)}$, then the inverse Laplace transform is

[$U(t-1)\,(1- e^{-t})$, $U(t-1)\,(1+ e^{-(t-1)})$, $U(t-1)\,(e^{-(t-1)})$, $U(t-1)\,(1- e^{-(t-1)})$]

30) If $F(s) = \dfrac{s}{s^2 - 4s + 8}$, then the inverse Laplace transform is

[$\frac{1}{2}\, e^{2t} \sin 2t$, $e^{2t} \cos 2t$, $\frac{1}{2}\, e^{2t} \cos 2t$, $e^{2t} (\cos 2t+\sin 2t)$, $e^{2t} (\cos 2t+2\sin 2t)$]

31) Let $f(t) = \begin{cases} 0 & 0 < t < 1 \\ t & t > 1 \end{cases}$

i-Write f(t) in terms of unit step function.

ii-Find the Laplace transform of f(t).

32) Let $f(t) = \begin{cases} t^2 + 1 & 0 \le t < 1 \\ e^{-3t} + 1 & 1 \le t < 2 \\ 1 & t \ge 2 \end{cases}$

i -Write f(t) in terms of unit step function.

Ii -Find the Laplace transform of f(t).

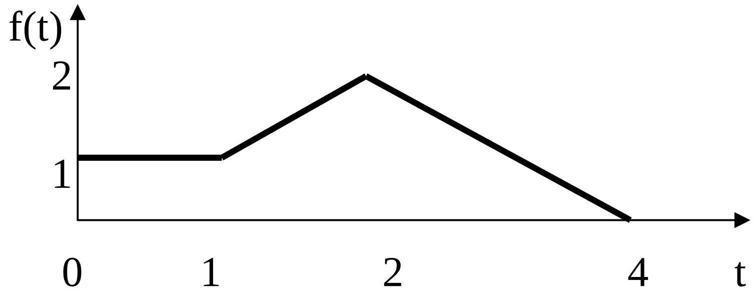
33) Let $f(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & t \le 0 \text{ or } t \ge 1 \end{cases}$

Solve $x'' + 4x = f(t)$, $x(0) = x'(0) = 0$

34) If $F(s) = \dfrac{e^{-2\pi s}}{(s+1)^2 + 4}$, then the inverse Laplace transform is

[$U(t-2\pi)\,(e^{-t}\sin 2t)$, $U(t-2\pi)\,(e^{-t}\sin 2t)/2$, $U(t-2\pi)\,(e^{-t+2\pi} \sin 2t)/2$]

35)



i-Write f(t) in terms of unit step function.

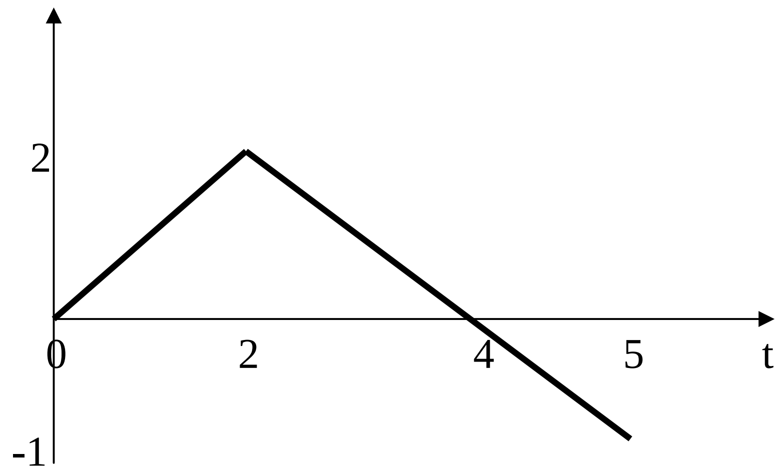
ii-Find the Laplace transform of f(t).

36) If $f(t) = \begin{cases} 6t-9 & 0 \leq t < 3 \\ t^2 & t \geq 3 \end{cases}$

i-Write f(t) in terms of unit step function.

ii-Find the Laplace transform of f(t).

37)



i-Write f(t) in terms of unit step function.

ii-Find the Laplace transform of f(t).

38)

Solve using Laplace Transform

$$y'' + 2y' + 5y = \sin(3t), \quad y(0) = 1 \quad y'(0) = -1$$

39) a) y` + y = 2U(t-1) – U(t-3) , y(0) = 2

b) Write formulas for y(t) in the indicated intervals:

0 < t < 1, then y(t) =….

1 < t < 3, then y(t) =….

3 < t , then y(t) =….

Special functions

I) Verify the following formulas

1) $\int_0^\infty \frac{t^{c-1}}{(t+b)(a-t)}dt = \frac{\pi}{a+b}[b^{c-1}\csc c\pi + a^{c-1}\cot c\pi]$

2) $\int_0^\infty \frac{(t+b)t^{d-1}}{(t+a)(t+c)}dt = \frac{\pi}{\sin d\pi}[\frac{a-b}{a-c}a^{d-1} + \frac{c-b}{c-a}c^{d-1}]$

3) $\int_0^\infty \frac{t^{c+1}}{(1+t^2)^2}dt = \frac{c\pi}{4\sin(c\pi/2)}$

4) $\int_0^\infty \frac{t^{ac-1}}{(1+t^c)^{a+b}}dt = \frac{1}{c}\beta(a,b)$

5) $\int_1^\infty \frac{dt}{(a-bt)(t-1)^c} = -\frac{\pi}{b}[\frac{b}{b-a}]^c \csc c\pi$

6) $\int_{-\infty}^a \frac{(a-t)^{p-1}}{t-b}dt = -\frac{\pi}{\sin p\pi}[b-a]^{p-1}$

7) $\int_1^\infty \frac{(t-1)^a}{t^b}dt = \beta(a+1, b-a-1)$

8) $\int_0^\infty \frac{t^{a-1}}{(1+ut)^{p+1}}dt = \frac{1}{u^a}\beta(a, p+1-a)$

9) $\int_0^1 t^{aq-1}(1-t^q)^{b-1}dt = \frac{1}{q}\beta(a,b)$

10) $\int_0^1 t^{p+q-1}(1-t^q)^{-p/q}dt = \frac{1}{q}\beta(1+p/q, 1-p/q)$

11) $\int_0^1 t^{q/p-1}(1-t^q)^{-1/p}dt = \frac{1}{q}\beta(1/p, 1-1/p)$

12) $\displaystyle\int_0^1 \frac{t^{aq-1}}{\sqrt[q]{(1-t^q)}}dt = \frac{1}{q}\beta(a,1-1/q)$

13) $\displaystyle\int_0^\infty \frac{t^{a-1}}{t+c}dt = \frac{\pi}{\tan(a\pi)}(-c)^{a-1}$

14) $\displaystyle\int_0^1 \frac{t^{3c-m}}{\sqrt[3]{(1-t^3)}}dt = \frac{1}{3}\beta(c+\frac{1-m}{3},\frac{2}{3})$

15) $\displaystyle\int_0^\infty \frac{a\,dt}{\sqrt{t}(a^2+t^2)} = \frac{\pi}{\sqrt{2a}}$

16) $\displaystyle\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}dx = \frac{1}{a^n b^m}\beta(m,n)$ and hence find the value $\displaystyle\int_0^\infty \frac{x^5}{(2+3x)^{16}}dx$

17) $\displaystyle\int_0^\infty x\, e^{-ax}\sin bx\, dx = \frac{2ab}{(a^2+b^2)^2}$ [Hint: put sinbx = Im. (e^{ibx})]

18) $\displaystyle\int_0^\infty x^{m-1}\cos ax\, dx = \frac{\Gamma(m)}{a^m}(\cos\frac{m\pi}{2})$ [Hint: put cosax = Re. (e^{iax})]

19) $\displaystyle\int_0^\infty x^{n-1}e^{-ax}\sin bx\, dx = \frac{\Gamma(n)}{(a^2+b^2)^{n/2}}\sin(n\tan^{-1}(\frac{b}{a}))$ [Hint: put sinbx = Im. (e^{ibx})]

20) $\displaystyle\int_a^b (x-a)^m(b-x)^n dx = (b-a)^{m+n+1}\beta(m+1,n+1)$, then deduce that:

$\displaystyle\int_5^9 \sqrt[4]{(x-5)(9-x)}dx = \frac{2(\Gamma(1/4))^2}{3\sqrt{\pi}}$ [Hint : put x-a = (b-a)t]

21) $\displaystyle\int_0^a \frac{dx}{(a^n-x^n)^{1/n}} = \frac{\pi}{n}\csc(\frac{\pi}{n})$

22) $\displaystyle\int_0^{\pi/2} (\tan x)^n dx = \frac{\pi}{2}\sec(\frac{n\pi}{2})$

23) $\int_0^{\pi/2} \frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}} = \frac{(\Gamma(1/4))^2}{4\sqrt{\pi}}$ [Hint: put tan(x/2) = t]

24) $\int_0^{\pi/2} (\sin x)^{2n}\, dx = \frac{\sqrt{\pi}}{2}\frac{\Gamma[(n+1)/2]}{n!}$

25) $\int_0^{\pi/2} (\sin\theta)^{P}\, d\theta = \int_0^{\pi/2} (\cos\theta)^{P}\, d\theta = \frac{\sqrt{\pi}}{2}\frac{\Gamma(\frac{p+1}{2})}{\Gamma(\frac{p}{2}+1)}$

II) Evaluate the following integrals

24) $\int_0^{2} \frac{t^2\, dt}{\sqrt{2-t}}$

25) $\int_{-1}^{1} (1-t^2)^{n}\, dt$,

26) $\int_0^{1/2} t^{m-1} \ln(1/2t)\, dt$,

27) $\int_0^{\infty} \frac{t^2\, dt}{1+t^4}$,

28) $\int_0^{\pi} \frac{dt}{\sqrt{3-\cos t}} = \frac{\Gamma(1/4)}{4\sqrt{\pi}}$ (Hint: put $\cos t = 1 - 2\sqrt{y}$)

29) $\int_0^{\pi/2} \cos^m t \sin^n t\, dt$

30) $\int_0^{3} \frac{dt}{\sqrt{3t-t^2}}$

31) $\int_0^{1} t^3 \sqrt[3]{8-t^3}\, dt$

32) $\int_0^{1} \frac{dt}{\sqrt{-\ln t}}$

33) $\int_0^\infty t^n e^{-mt} dt$

34) $\int_0^\infty x^n e^{-\sqrt{ax}} dx$

35) $\int_0^\infty \frac{x^n dx}{a^x}$

36) $\int_0^\infty a^{-mx^n} dx$

37) $\int_0^\infty \sqrt{x}\, e^{-\sqrt[3]{x}} dx$

38) $\int_0^\infty \frac{e^{-\sqrt{x}} dx}{x^{7/4}}$

39) $\int_0^1 x^4 (\log \frac{1}{x})^3 dx$

40) $\int_0^{\pi/4} (\cos 2\theta)^3 (\sin 4\theta)^4 d\theta$ [Hint: put $2\theta = t$]

41) $\int_0^{2\pi} (\sin\theta)^2 (1+\cos\theta)^4 d\theta$ [Hint: put $\theta/2 = t$]

42) $\int_{-\pi/4}^{\pi/4} (\sin\theta + \cos\theta)^{1/3} d\theta$ [Hint: $\int_{-\pi/4}^{\pi/4} [\sqrt{2}(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta)]^{1/3} d\theta =$

$\int_{-\pi/4}^{\pi/4} [\sqrt{2}\cos(\theta + \pi/4)]^{1/3} d\theta$, put $\theta + \pi/4 = t$]

43) $\int_{-\pi/6}^{\pi/3} (\sqrt{3}\sin\theta + \cos\theta)^{1/4} d\theta$

44) $\int_0^\infty \frac{x^8(1-x^6)\,dx}{(1+x)^{24}}$ [Hint: $\int_0^\infty \frac{x^8(1-x^6)\,dx}{(1+x)^{24}} = \int_0^\infty \frac{(x^8-x^{14})\,dx}{(1+x)^{24}}$]

45) $\int_0^\pi \frac{(\sin x)^{n-1}\,dx}{a+b\cos x}$ [Hint: put tan(x/2) = t]

46) Given $\left\lceil \frac{8}{5} \right.$ = 0.8935, find the value of $\left\lceil \frac{-12}{5} \right.$

47) If $\beta(n,2) = \frac{1}{42}$ and n is positive integer, find the value of n.

48) $\int_{-a}^{a} (a+x)^{m-1}(a-x)^{n-1}dx$

49) $\int_0^1 x^m (\log_a x)^n\,dx$

50) $\int_0^1 (-\log_a x)^{-1/2}\,dx$

51) $\int_0^\infty x^m e^{-ax^n}\,dx$

52) $\int_0^2 x\sqrt{(8-x^3)}dx$

III) Choose the correct answer

48) $\int_0^1 x^4 (\log \frac{1}{x})^3\,dx$ = (3/325, 6/625, 3/625, 6/325)

49) $\int_0^{\pi/2} \sqrt{\cot x}dx = (\sqrt{2}\pi/2, \pi/2, \pi/4, \sqrt{2}\pi/4)$

50) $\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-x^2/8}\,dx = (1, 2, \pi, 2\pi)$

51) $\int_0^\infty \sqrt{y}\, e^{-y^3}\, dy = (\sqrt{\pi}/2, \sqrt{\pi}/3, \sqrt{\pi}, \sqrt{\pi}/6)$

52) If $\beta(n,2) = \frac{1}{6}$ and n is positive integer, then the value of n is (3, -2, 2, -3)

53) The value of $\beta(m+1,n)$ is

$$\left(\frac{n}{m+n}\beta(m,n), \frac{n}{m+1}\beta(m,n), \frac{m}{m+n}\beta(m,n), \frac{m}{m+1}\beta(m,n)\right)$$

54) $\int_0^\infty \frac{t^2}{1+t^4} dt = (\pi/\sqrt{2}, \sqrt{\pi}/2, \pi/2, \pi/4)$

55) $\int_0^\infty e^{-y^3}\, dy = \left(\frac{1}{3}\Gamma\left(\frac{1}{3}\right), \frac{1}{3}\Gamma\left(\frac{1}{2}\right), \frac{1}{9}\Gamma\left(\frac{1}{3}\right), \frac{1}{3}\Gamma\left(\frac{2}{3}\right)\right)$

56) $\int_0^1 x^2(1-x)^{5/2} dx = (\beta(3,7/2), \beta(1,5/2), \beta(2,5/2), \beta(1,3/2))$

57) $\int_0^\infty x^4\, e^{-x^2}\, dx = (3\sqrt{\pi}/8, 3\sqrt{\pi}/4, \sqrt{\pi}/2, 15\sqrt{\pi}/8)$

58) $\int_0^1 \frac{dx}{\sqrt{1-x^4}} =$

$$\left(\Gamma(3/4)\Gamma(1/2), \Gamma(3/4)\Gamma(1/4), \frac{(1/4)\Gamma(3/4)\Gamma(1/2)}{\Gamma(5/4)}, \frac{(1/2)\Gamma(3/4)\Gamma(1/2)}{\Gamma(5/4)}\right)$$

59) Prove that :

$$\frac{\beta(m+1,n)}{m} = \frac{\beta(m,n+1)}{n} = \frac{\beta(m,n)}{m+n}$$

$$\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \int_0^{\pi/2} \sqrt{\sin\theta}\; d\theta = \pi$$

60) Match the items in columns I and II

a) $\beta(p,q)$ i) $\Gamma(1/2)$

b) $\dfrac{\Gamma p \Gamma q}{\Gamma(p+q)}$ ii) $\int_0^\infty \dfrac{x^{p-1}dx}{(1+x)^{p+q}}$

c) $\sqrt{\pi}$ iii) $\beta(p,q)$

d) $\dfrac{\pi}{\sin p\pi}$ iv) $\Gamma p \Gamma(1-p)$

Complex variables

Choose the correct answer

[Hint: A function f is said to be differentiable at α if: $\lim_{z\to\alpha} \frac{f(z) - f(\alpha)}{z - \alpha}$ exists;

and $\lim_{z\to\alpha} f(z) = f(\alpha)$, then f is said to be continuous at α].

1) $\lim_{z\to 0} \frac{\bar{z}}{z}$ is (0, 1, 1-i, doesn't exist)

2) f(z) = $z^2 + z$ is
(cont. only at z = 0, 1, not cont. anywhere, analytic everywhere)

3) f(z) = $\frac{1}{z}$ is continuous (at all points, only at z=0, except at z=0, nowhere)

4) f(z) = $\bar{z}$ is differentiable (<u>nowhere</u>, only at z=0, everywhere, only at z=1)

5) f(z) = $|z|^2$ is differentiable (<u>nowhere</u>, only at z=0, everywhere, only at z=1)

6) f(z) = $|z|^2$ is

a) Differentiable and analytic everywhere,

b) Not differentiable at z = 0, but analytic at z = 0,

c) Differentiable at z = 1 and not analytic at z = 1 only,

<u>d) Differentiable at z = 0 and not analytic at z = 1.</u>

7) $f(z) = \bar{z}$ is analytic (everywhere, <u>nowhere</u>, only at z=0, only at z=1)

8) $f(z) = z$ is (<u>analytic everywhere</u>, analytic nowhere, continuous but not analytic, analytic at z=0 only)

9) If $f(z) = \begin{cases} \dfrac{xy}{x^2+y^2} & z \neq 0 \\ 0 & z = 0 \end{cases}$, then f(z) is

a) Continuous but not differentiable at z = 0

b) Differentiable at z = 0

c) Analytic everywhere except at z = 0

d) <u>Not differentiable at z = 0</u>

10) $f(z) = e^z$ is analytic (only at z=0, only at z=i, nowhere, <u>everywhere</u>)

11) If $f(z) = \begin{cases} \dfrac{zx}{|z|} & z \neq 0 \\ 0 & z = 0 \end{cases}$, then f(z) is

a) Continuous and differentiable at z = 0

b) Not continuous but differentiable at z = 0

<u>c) Continuous but not differentiable at z = 0</u>

d) Everywhere continuous and differentiable.

12) $f(z) = z - \bar{z}$ is differentiable (everywhere, at z = 0 only, at z = 1 only, <u>nowhere</u>)

13) $f(z) = \text{Im. } z$ is differentiable(everywhere, at $z = 0$ only, at $z = 1$ only, nowhere)

14) $f(z) = \text{Re. } z$ is differentiable(everywhere, at $z = 0$ only, at $z = 1$ only, nowhere)

15) $f(z) = iz + 2$ is differentiable (everywhere, at $z = 0$ only, at $z = i$ only, nowhere)

16) $f(z) = x^2 - y^2 + 2ixy$ is differentiable(everywhere, at $z = 0$ only, at $z = 1$ only, nowhere)

17) $f(z) = 2z - \text{Im. } z$ is differentiable(everywhere, at $z = 0$ only, at $z = 1$ only, nowhere)

18) $f(z) = iz^2 - z$ is differentiable(everywhere, at $z = 0$ only, at $z = 1$ only, nowhere)

19) If $f(z) = x + ay + i(x+by)$ is differentiable at every point, then a and b values equal to (0 and 1, 1 and 2, -1 and 1, 2 and i)

20) If $f(z) = e^x\cos(ay) + i\,e^x\sin(y-b)$ is differentiable at every point, then a and b values equal to (0 and i, 1 and 0, 1 and 2, i and -i)

21) $f(z) = e^{-y}(ay\cos x - x\sin x) + i\,e^{-y}(-bx\cos x - y\sin x)$ is differentiable at every point, then a and b values equal to (1 and 0, -1 and -1, 1 and 2, i and 1)

22) $f(z) = e^x(\cos y - i\sin y)$ is (analytic, not analytic, analytic at $z=0$, analytic at $z=i$)

23) $f(z) = x^2 - y^2 - 2ixy$ is (analytic, not analytic, analytic at $z=0$, analytic at $z=1$)

24) If $f(z)$ is analytic, then $\overline{f(z)}$ is(analytic, not analytic, analytic at $z=0$, analytic at $z=1$)

25) If $g(z) = \cos\bar{z}$, then $\overline{g(z)}$ is (analytic, not analytic, analytic only at $z=0$, analytic only at $z=\pi/2$)

26) The harmonic conjugate of $x^2 - y^2 + y$ is ($2xy$, $2xy+x$, $2xy-x$, $x^2 - 2xy$)

27) The harmonic conjugate of $u = 2xy + 3y$ is ($x^2 - y^2$, $y^2 - 3x$, $x^2 + y^2 - 3y$, $y^2 - 3x - x^2$)

28) If $e^x\cos y$ is harmonic, then a = (± 1, 0, 1 and 2, i and 1)

29) The function $-x^3 + 3xy^2 + 2y + 1$ is (harmonic, not harmonic, analytic, constant)

30) The function cosax sinhy is harmonic if (a=1, a = i, a = 0, a = 2)

31) The harmonic conjugate of u = 4xy + x + 1is ($2x^2 - y^2 + y + c$, $2y^2 - y + c$, $y^2 - x^2 + c$, $2y^2 + y - 2x^2 + c$)

32) f(z) = $\frac{a}{2} r\cos\theta + i(r\sin\theta + 2)$ is harmonic if (a=0, a = 1, a = 2, a = 3)

33) If f(z) = u+iv in polar form, then $\frac{\partial u}{\partial r} = (\frac{\partial v}{\partial \theta}, r\frac{\partial v}{\partial \theta}, \frac{1}{r}\frac{\partial v}{\partial \theta}, -\frac{\partial v}{\partial \theta})$

34) If f(z) = u+iv in polar form, then $\frac{\partial u}{\partial \theta} = (\frac{\partial v}{\partial r}, -\frac{1}{r}\frac{\partial v}{\partial r}, -\frac{\partial v}{\partial r}, -r\frac{\partial v}{\partial r})$

Fourier series

1) Expand in Fourier series the following functions

a) $f(x) = x + x^2$, $-2 < x < 2$

b) $f(x) = |x|$, $-3 < x < 3$

c) $f(x) = x^2$, $-\pi < x < \pi$, then deduce the sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$

d) $f(x) = \begin{cases} \pi/2 + x, & -\pi \le x \le 0 \\ \pi/2 - x, & 0 < x \le \pi \end{cases}$, then deduce the sum

$$\frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \ldots.$$

e) $f(x) = |\sin x|$, $0 < x < 2\pi$, then deduce the sum $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots.$

f) $f(x) = |\cos x|$, $-\pi < x < \pi$, then deduce the sum

$$\frac{1}{1^2.3^2}+\frac{1}{3^2.5^2}+\frac{1}{5^2.7^2}+....$$

g) $f(x) = x\,|x|$, $-\pi < x < \pi$

h) $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ x/\pi, & 0 < x \le \pi \end{cases}$,

i) $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ -(x-\pi), & \pi < x \le 2\pi \end{cases}$

j) $f(x) = \begin{cases} 0 & -1 \le x \le -1/2 \\ \cos 3\pi x & -1/2 < x \le 1/2 \\ 0 & 1/2 \le x < 1 \end{cases}$, $f(x+2) = f(x)$

k) $f(x) = e^x$, $-\pi < x < \pi$, use the result to find the sum of series $\sum_{n=2}^{\infty} \frac{(-1)^n}{1+n^2}$

L) $f(x) = \begin{cases} x, & 0 < x \le \pi \\ x+\pi, & -\pi < x \le 0 \end{cases}$

2) Expand in half range cosine (cosine harmonic) the function

a) $f(x) = \sin x$, $0<x<\pi$, b) $f(x) = x$, $0< x< 1$, c) $f(x) = e^x$, $0<x<\pi$

d) $f(x) = x(\pi - x)$, $0 < x < \pi$, then deduce the sum $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

3) Expand in half range sine (sine harmonic) the following functions

a) $f(x) = x^2$ $0< x< 2$, b) $f(x) = \cos x$ $0<x<\pi$,

c) $f(x) = x(\pi - x)$, $0 < x < \pi$, then deduce the sum $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}$

4) Expand (2)and (3) in odd harmonic and in Even harmonic

5) Expand $f(x) = x$, $0< x < 1$ in

a) Even cosine harmonic
b) Odd cosine harmonic

c) Even sine harmonic
d) Odd sine harmonic

6) Expand in complex Fourier

a) $f(x) = e^x$, $-\pi < x < \pi$, b) $f(x) = x$, $-1 < x < 1$

7) Expand the function $f(x) = x$, $0 < x < T$ in

a) Fourier cosine
b) Fourier sine
c) Odd harmonic
d) Even harmonic
e) Even Cosine harmonic
f) Even Sine harmonic

8) Find Fourier transforms of the following functions

a) $f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$, then evaluate $\int_0^{\infty} \left(\frac{x\cos x - \sin x}{x^3}\right)\cos(x/2)\,dx$

b) $f(x) = \begin{cases} 1 & 0 < x < a \\ -1 & -a < x < 0 \\ 0 & |x| > a \end{cases}$

9) Expand the following functions in Fourier sine series:

a) $f(x) = \begin{cases} \pi/2 - x, & 0 \le x \le \pi/2 \\ 0, & \pi/2 < x \le \pi \end{cases}$,

b) $f(x) = e^x$, $0 < x < 1$

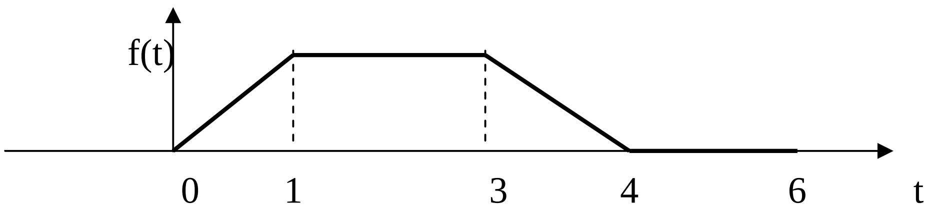
10) If $f(x) = \begin{cases} 0 & x \le -\pi/2 \\ \pi/2 + x & -\pi/2 < x \end{cases}$, then the first Fourier cosine coefficient equal …..and the second Fourier sine coefficient equal …..

11) Find the first Fourier cosine coefficients of the functions:

a) $f(x) = \begin{cases} x-5 & 0<x\le 5 \\ 0 & 5<x\le 10 \end{cases}$, b) $f(x) = \begin{cases} 0 & 0<x\le 1 \\ 1-x & 1<x\le 2 \end{cases}$,

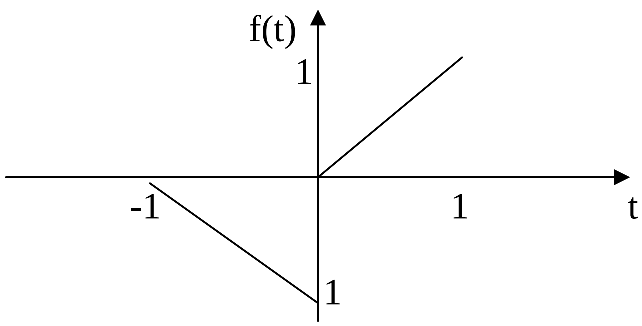
12) Expand the following functions in Fourier series:

a)



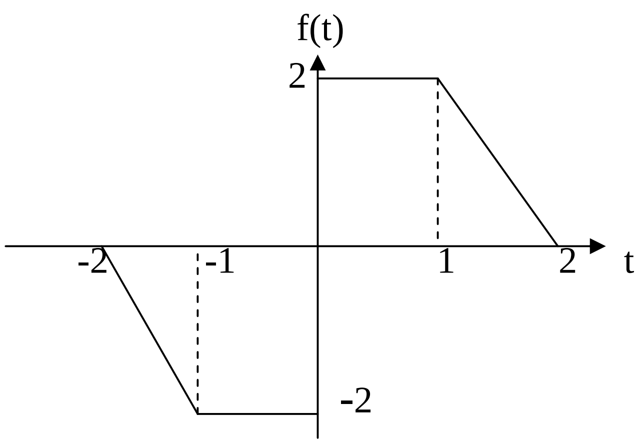
$f(t+6) = f(t)$

b)



$f(t+2) = f(t)$

c)



$f(t+4) = f(t)$

13) Suppose f(x) = $\pi x^2 - 2x^3$ on $[0, \pi]$ and g(x) is the sum of the whole Fourier sine series for f(x) and h(x) is the sum of the whole Fourier cosine series for f(x), compute g(1), h(1), g(π), h(π).

14) Prove: x = $\frac{4}{\pi}[\sin x - \frac{\sin 3x}{9} + \frac{\sin 5x}{25} - \ldots]$ in the interval$[-\frac{\pi}{2}, \frac{\pi}{2}]$.

15) Find the Fourier series of the function f defined by

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases},$$

and f has period 2π. What does the Fourier series converge to at x = 0?

16) Find the Fourier series of the function f defined by f(x) = $\begin{cases} 1, & -\pi < x < 0 \\ 3, & 0 < x < \pi \end{cases}$,

and f has period 2π. What does the Fourier series converge to at x = 0?

17) Let h be a given number in the interval $(0, \pi)$. Find the Fourier cosine series of

the function f(x) = $\begin{cases} 1, & 0 < x < h \\ 0, & h < x < \pi \end{cases}$,

18) Calculate the Fourier sine series of the function defined by f(x) = $x(\pi - x)$

on $(0, \pi)$. Use its Fourier representation to find the value of the infinite series

1- $\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\frac{1}{9^3}-....$

19) Let h be a given number in the interval (0, π). Find the Fourier cosine series representation of f(x) = $\begin{cases} 1 & 0 < x < a \\ \frac{2h-x}{2h} & 0 < x < 2h \\ 0 & 2h < x < \pi \end{cases}$

20) What is the Fourier sine series and Fourier cosine series of f(x) = π/4 - x/2, where 0 < x < π.

21) Find the Fourier series of f(x) = |x| where −L < x < L.

22) The function f is defined by f(x) = e^x for −L < x < L. Find its Fourier series.

23) Let a be a given integer. The function f is defined by f(x) = sinax for 0 < x < π. Find its Fourier cosine series.

24) Let f be a periodic function of period 2 π such that f(x) = $\pi^2 - x^2$ for

$x \in (-\pi, \pi)$. Supposing that f has a convergent trigonometric Fourier series, show

that $\pi^2/12 = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$.

25) Find the Fourier series of f(x) = 3x for $x \in (-\pi, \pi)$, then deduce the sum

1- $\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+....$

26) Let f(x) be a function of period 2 π such that f(x) = $\begin{cases} x, & 0 < x < \pi \\ \pi, & \pi < x < 2\pi \end{cases}$, Sketch a graph of f(x) in the interval −2 π < x < 2 π and find the sum of

$1- \frac{1}{3}+\frac{1}{5}-\frac{1}{7}+....$ and $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+....$

27) Find the Fourier series of f(x) = x/2 for x $\in(-\pi, \pi)$, then deduce the sum

$1- \frac{1}{3}+\frac{1}{5}-\frac{1}{7}+....$

28) Let f(x) be a function of period 2 π such that f(x) = $\begin{cases} \pi - x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$,

Sketch a graph of f(x) in the interval −2 π < x < 2 π and find the sum of

$1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+....$

29) Find the Fourier series of f(x) = x^2 for x $\in(-\pi, \pi)$, then deduce the sum

$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+....$

30) Let the 2 π periodic function f (x) be specified on the interval (−π, π) by f (x) = x^2.

(a) Determine the coefficients $\{c_k\}$ and $\{d_k\}$ in the Fourier Series expansion

f(x) = $\sum_{k=1}^{\infty} c_k \cos kx + \sum_{k=1}^{\infty} d_k \sin kx$, then determine the value of $\sum_{k=1}^{\infty} \frac{1}{k^4}$

31) Determine the Fourier series to $f(x) = e^{-|x|}$, $-\pi < x < \pi$.

32) Determine the Fourier series to $f(x) = \sin(3x/2)$, $-\pi < x < \pi$.

33) Determine the Fourier series to $f(x) = x \sin(x)$, $-\pi < x < \pi$.

34) $f(x) = e^{-4x}$ for $-2 < x < 2$ with $f(x + 4) = f(x)$.

35) $f(t) = \begin{cases} \pi^2, & -\pi < t < 0 \\ (t-\pi)^2, & 0 < t < \pi \end{cases}$, with $f(t) = f(t + 2\pi)$. Show how to use this Fourier series to compute the sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$

36) Expand in Fourier series

a) $f(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi/2 & \pi/2 < x < \pi \\ \pi - x/2 & \pi < x < 2\pi \end{cases}$, with $f(x) = f(x + 2\pi)$

b) $f(x) = \begin{cases} \sin(t/2), & 0 < t < \pi \\ -\sin(t/2), & \pi < t < 2\pi \end{cases}$, with $f(t) = f(t + 2\pi)$

Probability

1) The student body of a large university consists of 60% female students. A random sample of 8 students is selected.

(A) What is the probability that among the students in the sample exactly two are female?
a. 0.0896
b. 0.2936
c. 0.0413
d. 0.0007
e. None of the above

(B) What is the probability that among the students in the sample at least 6 are male?
a. 0.0413
b. 0.0079
c. 0.0007
d. 0.0499
e. None of the above

2) The credit card balance of one elderly group is normally distributed with a

mean of \$200 and a standard deviation of \$25.

(A) The probability of an elderly person from this group having a credit card balance of more than \$241.25 is
a. 0.0252
b. 0.0495
c. 0.0901
d. 0.9010
e. None of the above

(B) What percent of elderly people in this group would we expect to have balances between \$180 and \$220?
a. 28.81%
b. 45.88%
c. 57.62%
d. 64.23%
e. None of the above

3) The starting debt balances for new dentists are normally distributed with a mean of \$40,000 and a standard deviation of \$5,000.

(A) What is the probability that a randomly selected new dentist will have a starting debt balance of at least \$47,500?
a. 0.1534
b. 0.7564
c. 0.0668
d. 0.1053
e. None of the above

(B) What percentage of new dentists will have starting debt balances between \$34,000 and \$46,000?
a. 38.49%

b. 39.59%
c. 69.76%
d. 76.98%
e. None of the above

4) A particular county in Louisiana experienced incidents of West Niles virus at an average rate of 2.6 per month.

(A) What is the probability of at least three persons coming down with West Niles virus during a month?
a. 12.26%
b. 21.76%
c. 26.40%
d. 48.16%
e. None of the above

(B) What is the probability of an incident occurring every 15 days or less (assume a 30-day month)?
a. 27.25%
b. 45.53%
c. 65.67%
d. 72.75%
e. None of the above

5) A random sample of 15 people is taken from a population in which 40% favour a particular political stand. What is the probability that exactly 6 individuals in the sample favour this political stand?

(a) 0.4000
(b) 0.5000
(c) 0.4000

(d) 0.2066
(e) 0.0041

6) Experience has shown that a certain lie detector will show a positive reading (indicates a lie) 10% of the time when a person is telling the truth and 95% of the time when a person is lying. Suppose that a random sample of 5 suspects is subjected to a lie detector test regarding a recent one-person crime. Then the probability of observing no positive reading if all suspects plead innocent and are telling the truth is

(a) 0.409
(b) 0.735
(c) 0.00001
(d) 0.591
(e) 0.99999

7) It has been estimated that about 30% of frozen chicken contain enough salmonella bacteria to cause illness if improperly cooked. A consumer purchases 12 frozen chickens. What is the probability that the consumer will have more than 6 contaminated chickens?

(a) .961
(b) .118
(c) .882
(d) .039
(e) .079

8) Refer to the previous question. Suppose that a supermarket buys 1000 frozen chickens from a supplier. Find an approximate 95% interval for the number of frozen chickens that may be contaminated.

(a) (90, 510)
(b) (285, 315)

(c) (0, 730)
(d) (270, 330)
(e) (255, 345)

9) Which of the following is NOT an assumption of the Binomial distribution?

(a) All trials must be identical.
(b) All trials must be independent.
(c) Each trial must be classified as a success or a failure.
(d) The number of successes in the trials is counted.
(e) The probability of success is equal to .5 in all trials.

10) It has been estimated that as many as 70% of the fish caught in certain areas of the Great Lakes have liver cancer due to the pollutants present. Find an approximate 95% range for the number of fish with liver cancer present in a sample of 130 fish.

(a) (80, 102)
(b) (86, 97)
(c) (63, 119)
(d) (36, 146)
(e) (75, 107)

11) In a triangle test a tester is presented with three food samples, two of which are alike, and is asked to pick out the odd one by testing. If a tester has no well developed sense and can pick the odd one only, by chance, what is the probability that in five trials he will make four or more correct decisions?

(a) 11/243
(b) 1/243
(c) 10/243
(d) 233/243
(e) 232/243

12) The probability that a certain machine will produce a defective item is 1/4. If a random sample of 6 items is taken from the output of this machine, what is the probability that there will be 5 or more defectives in the sample?

(a) 1/4096

(b) 3/4096

(c) 4/4096

(d) 18/4096

(e) 19/4096

13) The probability that a certain machine will produce a defective item is 0.20. If a random sample of 6 items is taken from the output of this machine, what is the probability that there will be 5 or more defectives in the sample?

(a) .0001

(b) .0154

(c) .0015

(d) .2458

(e) .0016

14) Suppose 60% of a herd of cattle is infected with a particular disease. Let Y = the number of non-diseased cattle in a sample of size 5. The distribution of Y is

(a) binomial with n = 5 and p = 0.6

(b) binomial with n = 5 and p = 0.4

(c) binomial with n = 5 and p = 0.5

(d) the same as the distribution of X, the number of infected cattle.

(e) Poisson with $\lambda = .6$

15) Fifteen percent of new residential central air conditioning units installed by a supplier need additional adjustments requiring a service call. Assume that a recent sample of seven such units constitutes a Bernoulli process. Interest centers on X, the number of units among these seven that need additional adjustments. The mean and variance of X are, respectively

(a) .15; .85

(b) .15; 1.05

(c) .15; .8925

(d) 1.05; .1275

(e) 1.05; .8915

16) If you buy one ticket in the Provincial Lottery, then the probability that you will win a prize is 0.11. If you buy one ticket each month for five months, what is the probability that you will win at least one prize?

(a) 0.55

(b) 0.50

(c) 0.44

(d) 0.45

(e) 0.56

17) Suppose that the probability that a cross between two varieties will express a particular gene is 0.20. What is the probability that in 8 progeny plants, two or fewer plants will express the gene?

(a) .2936

(b) .3355

(c) .1678

(d) .6291

(e) .7969

18) Refer to the previous question. Suppose that 120 crosses are bred. Find

a likely 95% range for the number of progeny that will express the gene.

(a) 24s19.2

(b) 24s4.4

(c) 24s8.8

(d) 24s4.9

(e) 24s9.8

19) Seventeen people have been exposed to a particular disease. Each one independently has a 40% chance of contracting the disease. A hospital has the capacity to handle 10 cases of the disease. What is the probability that the hospital's capacity will be exceeded?

(a) .965

(b) .035

(c) .989

(d) .011

(e) .736

20) Refer to the previous problem. Planners need to have enough beds available to handle a proportion of all outbreaks. Suppose a typical outbreak has 100 people exposed, each with a 40% chance of coming down with the disease. Which is not correct?

(a) This experiment satisfies the assumptions of a binomial distribution.

(b) About 95% of the time, between 30 and 50 people will contract the disease.

(c) Almost all of the time, between 25 and 55 people will contract the disease.

(d) On average, about 40 people will contract the disease.

(e) Almost all of time, less than 40 people will be infected.

21) There are 10 patients on the Neo-Natal Ward of a local hospital who are monitored by 2 staff members. If the probability (at any one time) of a patient requiring emergency attention by a staff member is .3, assuming the patients to be behave independently, what is the probability at any one time that there will not be sufficient staff to attend all emergencies?

(a) .3828

(b) .3000

(c) .0900

(d) .9100

(e) .6172

22) A newborn baby whose Apgar score is over 6 is classified as normal and this happens in 80% of births. As a quality control check, an auditor examined the records of 100 births. He would be suspicious if the number of normal births in the

sample of 100 births fell above the upper limit of a “95%-normal-range”. What is this upper limit?

(a) 112

(b) 72

(c) 88

(d) 8

(e) none of these

23) Refer to the previous question. Babies that have Apgar scores of 6 or lower require more expensive medical care. What is the probability that in the next 10 births, 3 or more babies will have Apgar scores of 6 or lower?

(a) .2013

(b) .3222

(c) .9999

(d) .0001

(e) .1536

24) Newsweek in 1989 reported that 60% of young children have blood lead levels that could impair their neurological development. Assuming that a class in a school is a random sample from the population of all children at risk, the

probability that at least 5 children out of 10 in a sample taken from a school may have a blood level that may impair development is:

(a) about .25

(b) about .20

(c) about .84

(d) about .16

(e) about .64

25) Refer to the previous problem. The total number of children in the school is about 400. In order to estimate the cost of treating all the children at one school, the health board wishes to be reasonably sure of the upper limit on the number of children affected. This upper limit is:

(a) about 260

(b) about 350

(c) about 240

(d) about 400

(e) about 250

26) Consider 8 blood donors chosen randomly from a population. The probability that the donor has type A blood is .40. Which of the following is correct?

(a) The probability of 1 or fewer donors having type A blood is about.11.

(b) The probability of 7 or more donors NOT having type A blood is about .0087.

(c) The probability of exactly 5 donors having type A blood is about .28.

(d) The probability of exactly 5 donors NOT having type A blood is about .12.

(e) The probability that between 3 and 5 donors (inclusive) will have type A blood is about .37.

27) Consider 100 blood donors chosen randomly from a population where the probability of type A is 0.40? What is the approximate probability that at least 43 donors will have type A blood?

(a) about .43

(b) about .62

(c) about .73

(d) about .27

(e) about .38

28) It is sometimes possible to obtain approximate probabilities associated with values of a random variable by using the probability distribution of a different random variable. For example, binomial probabilities using the Poisson probability function, binomial probabilities using the normal etc. In order for the Poisson to give "good" approximate values for binomial probabilities we must have the condition(s) that:

(a) the population size is large relative to the sample size.

(b) the sample size is large

(c) the probability, p, is small and the sample size is large

(d) the probability, p, is close to .5 and the sample size is large

(e) the probability, p, is close to .5 and the population size is large

29) Suppose flaws (cracks, chips, specks, etc.) occur on the surface of glass

with density of 3 per square metre. What is the probability of there being

exactly 4 flaws on a sheet of glass of area 0.5 square metre?

(a) 0.047

(b) 0.168

(c) 0.981

(d) 0.815

(e) 0.647

30) The rate at which a particular defect occurs in lengths of plastic film being produced by a stable manufacturing process is 4.2 defects per 75 meter length. A random sample of the film is selected and it was found that the length of the film in the sample was 25 meters. What is the probability that there will be at most 2 defects found in the sample?

(a) .2102

(b) .2417

(c) .8335

(d) .1323

(e) .1665

31) Refer to the previous question. The manufacturer decides to examine a larger amount of film. She selects 1000 m of film. If there were no change in the defect rate from the old process, what would be the number of defects seen in approximately 95% of such examinations?

(a) (49 to 63)

(b) (34 to 78)

(c) (62 to 98)

(d) (41 to 71)

(e) (71 to 89)

32) The number of traffic accidents per week in a small city has a Poisson distribution with mean equal to 1.3. What is the probability of at least two accidents in 2 weeks?

(a) 0.2510

(b) 0.3732

(c) 0.5184

(d) 0.7326

(e) 0.4816

33) The number of traffic accidents per week in a small city has Poisson distribution with mean equal to 3. What is the probability of at least one accident in 2 weeks?

(a) 0.0174

(b) 0.9502

(c) 0.9975

(d) 0.1991

(e) 0.0025

34) Significant birth defects occur at a rate of about 4 per 1000 births in human populations. After a nuclear accident, there were 10 defects observed in the next 1500 births. Find the probability of observing at least 10 defects in this sample if the rate had not changed after the accident.

(a) .008

(b) .003

(c) .041

(d) .084

(e) .042

35) Refer to the previous question. An approximate 95% interval for the number of defects that would occur in 1500 births (assuming that the rate has not changed) is:

(a) (4, 8)

(b) (2, 10)

(c) (2, 6)

(d) (0, 8)

(e) (0, 12)

36) In a certain communications system, there is an average of 1 transmission error per 10 seconds. Let the distribution of transmission errors be Poisson. What is the probability of more than 1 error in a communication one-half minute in duration?

(a) 0.950

(b) 0.262

(c) 0.738

(d) 0.199

(e) 0.801

37) Bacteria in hamburger are distributed through out the meat. Suppose that a large batch of hamburger has an average contamination of 0.3 bacteria/ gram. Then the probability that a 10 gram sample will contain one or fewer bacteria is:

(a) .2222

(b) .7408

(c) .9603

(d) .1494

(e) .1992

38) Refer to the previous question. A 95% range for the likely number of bacteria present in a 100 g sample is:

(a) 30s30.0

(b) 30s5.5

(c) 30s11.0

(d) 30s16.4

(e) 30s2.8

39) The number of bacteria in a drop of water from a lake has a Poisson distribution with an average of 0.5 bacteria/drop. A small dish containing four drops of water from the lake is placed under a microscope. The probability of observing at most one bacteria in the sample is

(a) 0.910

(b) 0.406

(c) 0.271

(d) 0.135

(e) 0.303

40) Refer to the previous question. An approximate 95% range for the number of bacteria present in 400 drops of water is:

(a) (171,229)

(b) (361,439)

(c) (185,215)

(d) (157,243)

(e) (0,400)

41) Which of the following is NOT applicable to a Poisson Distribution?

(a) It is used to compute the probability of rare events.

(b) Every event is independent of every other event.

(c) It is parameterized by the sample size and the probability that an event will occur.

(d) The theoretical range for the number of events that could occur is 0,1,2,3, ...

(e) In order to compute the parameter value, we need to know the standardized rate and the sample size.

42) In a biological cell the average member of genes that will change into mutant genes, when treated radioactively, is 2.4. Assuming Poisson probability distribution find the probability that there are at most 3 mutant genes in a biological cell after the radioactive treatment.

(a) .2090

(b) .7576

(c) .5697

(d) .7787

(e) 1.000

43) The number of telephone calls that pass through a switchboard has a Poisson distribution with mean equal to 2 per minute. The probability that no telephone calls pass through the switch board in two consecutive minutes is:

(a) 0.2707

(b) 0.0517

(c) 0.0183

(d) 0.0366

(e) 0.1353

44) The distribution of phone calls arriving in one minute periods at a switchboard is assumed to be Poisson with the parameter λ. During 100 periods, the following distribution was obtained:

# (calls)	0	1	2	3	4 or more

Frequency	30	43	21	6	0

An estimate for λ based on this data set is:

(a) 1.00

(b) 1.03

(c) 1.04

(d) 1.33

(e) 1.37

45) A can company reports that the number of breakdowns per 8-hour shift on its machine-operated assembly line follows a Poisson distribution with a mean of 1.5. Assuming that the machine operates independently across shifts, what is the probability of no breakdowns during three consecutive 8-hour shifts?

(a) .0744

(b) .0498

(c) .6065

(d) .2231

(e) .0111

46) A fisherman arrives at his favorite fishing spot. From past experience he knows that the number of fish he catches per hour follows a Poisson distribution at 0.5 fish/hour. The probability that he catches at least 3 fish in four hours is:

(a) .0126

(b) .0144

(c) .1804

(d) .3233

(e) .8571

47) The number of arrivals per hour at an automatic teller machine is Poisson distributed with a mean of 3.5 arrivals/hour. What is the probability that more than three arrivals occur in an hour?

(a) .3209

(b) .4633

(c) .5367

(d) .6791

(e) .7246

48) The marketing manager of a company has noted that she usually receives 10 complaint calls during a week (consisting of five working days), and that the calls occur at random. Let us suppose that the number of calls during a week follows the Poisson distribution. The probability that she gets five such calls in one day is:

(a) .0361

(b) .0378

(c) .9834

(d) .2000

(e) .5

49) Cataracts are a very rare birth defect. In Canada, they occur at a rate of approximately 3 babies in every 100,000 births. In 1989, there were approximately 57,000 births in BC. The probability that more than 5 babies will be born with cataracts is approximately:

(a) about .1080

(b) about .0295

(c) about .0216

(d) about .0080

(e) about .0839

50) The number of deaths due to stroke in the Vancouver region each year varies randomly with a mean of about 555 deaths per year. Assuming that the number of deaths has an approximate Poisson distribution, then the probability that there will be at least 600 deaths due to stroke in any one year is:

(a) about 1%

(b) about 32%

(c) about 16%

(d) about 5%

(e) about 2.5%

51) The number of babies born with a particular severe eye defect each year varies randomly, but at a rate of about 30/10,000 live births. Last year there were about 15,000 live births. The approximate probability that there will be more than 58 babies born with this eye defect is:

(a) about 16%

(b) about 5%

(c) about 1%

(d) about 0.5%

(e) about 2.5%

52) If x is the number of successes in an independent series of 10 Bernoulli trials, then x has a _______________ distribution.

hypergeometric
Poisson
normal
binomial
exponential

53) Twenty five items are sampled. Each of these has the same probability of being defective. The probability that exactly 2 of the 25 are defective could best be found by _______.

using the normal distribution
using the binomial distribution
using the Poisson distribution
using the exponential distribution
using the uniform distribution

54) A fair coin is tossed 5 times. What is the probability that exactly 2 heads are observed?

0.313
0.073
0.400
0.156
0.250

55) A student randomly guesses the answers to a five question true/false test. If there is a 50% chance of guessing correctly on each question, what is the probability that the student misses no questions?

0.000
0.200
0.500
0.031
1.000

56) The number of cars arriving at a toll booth in five-minute intervals is Poisson distributed with a mean of 3 cars arriving in five-minute time intervals. The probability of 5 cars arriving over a five-minute interval is _______.

0.0940
0.0417
0.1500
0.1008
0.2890

57) For the Poisson distribution of a random variable lambda (λ) is 5 occurrences per ten-minute time interval. If we want to analyze the number of occurrences per hour, we must use an adjusted value for lambda equal to _______.

5
60
30
10
20

58) If x, the time (in minutes) to complete an oil change job at certain auto service station, is uniformly distributed over the interval 20 to 30, inclusively ($20 \le x \le 30$), then the mean of this distribution is ___________________.

50
25
10
15
5

59) The difference between a random variable and a probability distribution is
A) A random variable does not include the probability of an event
B) A random variable can only assume whole numbers
C) A probability distribution can only assume whole numbers
D) None of the above.

60) Which of the following is not a requirement of a binomial distribution?
A) A constant probability of success.
B) Only two possible outcomes.
C)A fixed number of trails.
D)Equally likely outcomes.

61) The mean and the variance are equal in\
A) All probability distributions.
B) The binomial distribution.
C) The Poisson distribution

D) The hypergeometric distribution.

62) In which of the following distributions is the probability of a success usually small?

A) Binomial
B) Poisson
C) Hypergeometric
D) All distribution

63) Which of the following is not a requirement of a probability distribution?

A) Equally likely probability of a success.
B) Sum of the possible outcomes is 1.00.
C) The outcomes are mutually exclusive.
D) The probability of each outcome is between 0 and 1

64) For a binomial distribution

A) n must assume a number between 1 and 20 or 25.
B) π must be a multiple of .10.
C) There must be at least 3 possible outcomes.
D) None of the above.

65) Which of the following is a major difference between the binomial and the hypergeometric distributions?

A) The sum of the outcomes can be greater than 1 for the hypergeometric
. **B)** The probability of a success changes from trial to trial in the hypergeometric distribution.
C) The number of trials changes in the hypergeometric distribution
D) The outcomes cannot be whole numbers in the hypergeometric distribution.

66) In a continuous probability distribution

A) Only certain outcomes are possible
B) All the values within a certain range are possible
C) The sum of the outcomes is greater than 1.00
D) None of the above.

67) For a binomial distribution with n = 15 as π changes from .50 toward .05 the distribution will

A) Become more positively skewed
. **B)** Become more negatively skewed
C) Become symmetrical
. **D)** All of the above.

68) The expected value of the a probability distribution
A) Is the same as the random variable.
B) Is another term for the mean
C) Is also called the variance.
D) Cannot be greater than 1.

69) Which of the following is an experiment?

Tossing a coin.

Rolling a single 6-sided die.

Choosing a marble from a jar.

All of the above.

70) Which of the following is an outcome?

- Rolling a pair of dice.
- Landing on red.
- Choosing 2 marbles from a jar.
- None of the above.

71) Which of the following experiments does NOT have equally likely outcomes?

- Choose a number at random from 1 to 7.
- Toss a coin.
- Choose a letter at random from the word SCHOOL.
- None of the above.

72) What is the probability of choosing a vowel from the alphabet?

- 21/26
- 5/26
- 1/21
- None of the above.

73) A number from 1 to 11 is chosen at random. What is the probability of choosing an odd number?

- 1/11
- 5/11
- 6/11
- None of the above.

74) Spin a spinner numbered 1 to 7, and toss a coin. What is the probability of getting an odd number on the spinner and a tail on the coin?

- 3/14
- 2/7
- 5/14
- None of the above.

75) A jar contains 6 red balls, 3 green balls, 5 white balls and 7 yellow balls. Two balls are chosen from the jar, with replacement. What is the probability that both balls chosen are green?

- 6/441
- 2/49
- 1/49
- None of the above

76) In Exercise 2, what is the probability of choosing a red and a yellow ball?

- 2/21
- 3/21
- 13/63
- All of the above.

77)Four cards are chosen from a standard deck of 52 playing cards with replacement. What is the probability of choosing 4 hearts in a row?

- 13/256
- 1/16
- 1/256
- None of the above.

78)A nationwide survey showed that 65% of all children in the United States dislike eating vegetables. If 4 children are chosen at random, what is the probability that all 4 dislike eating vegetables? (Round your answer to the nearest percent.)

- 18%

- 260%
- 2%
- None of the above.

79) What is the sample space for choosing an odd number from 1 to 11 at random?

- 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
- {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}
- {1, 3, 5, 7, 9 11}
- None of the above.

80) What is the sample space for choosing a prime number less than 15 at random?

- {2, 3, 5, 7, 11, 13, 15}
- {2, 3, 5, 7, 11, 13}
- {2, 3, 5, 7, 9, 11, 13}
- All of the above.

81) What is the sample space for choosing 1 jelly bean at random from a jar containing 5 red, 7 blue and 2 green jelly beans?

- {5, 7, 2}
- {5 red, 7 blue, 2 green}
- {red, blue, green}

- None of the above.

82) What is the sample space for choosing 1 letter at random from 5 vowels?

- {a, e, i, o, u}
- {v, o, w, e, l}
- {1, 2, 3, 4, 5}
- None of the above.

83) What is the sample space for choosing 1 letter at random from the word DIVIDE?

- {d, i, v, i, d, e}
- {1, 2, 3, 4, 5, 6}
- {d, i, v, e}
- None of the above.

84)Two cards are chosen at random from a deck of 52 cards without replacement.

What is the probability that the first card is a jack and the second card is a ten?

- 3/676
- 1/169
- 4/663

None of the above.

85)On a math test, 5 out of 20 students got an A. If three students are chosen at random without replacement, what is the probability that all three got an A on the test?

- 1/114
- 25/1368
- 3/400
- None of the above.

86)Three cards are chosen at random from a deck of 52 cards without replacement. What is the probability of choosing an ace, a king, and a queen in order?

- 1/2197
- 8/5525
- 8/16.575
- None of the above.

87)A school survey found that 7 out of 30 students walk to school. If four students are selected at random without replacement, what is the probability that all four walk to school?

- 343/93.960
- 1/783
- 7/6750
- None of the above.

88) Two cards are chosen at random from a deck of 52 cards without replacement. What is the probability of choosing two kings?

- 4/663
- 1/221
- 1/69
- None of the above.

89)In New York State, 48% of all teenagers own a skateboard and 39% of all teenagers own a skateboard and roller blades. What is the probability that a teenager owns roller blades given that the teenager owns a skateboard?

- 87%
- 81%
- 123%
- None of the above.

90) At a middle school, 18% of all students play football and basketball and 32% of all students play football. What is the probability that a student plays basketball given that the student plays football?

- 56%
- 178%
- 50%
- None of the above.

91) In the United States, 56% of all children get an allowance and 41% of all children get an allowance and do household chores. What is the probability that a child does household chores given that the child gets an allowance?

- 137%
- 97%
- 73%
- None of the above.

92)In Europe, 88% of all households have a television. 51% of all households have a television and a VCR. What is the probability that a household has a VCR given that it has a television?

- 173%
- 58%
- 42%
- None of the above.

93) In New England, 84% of the houses have a garage and 65% of the houses have a garage and a back yard. What is the probability that a house has a backyard given that it has a garage?

- 77%
- 109%
- 19%
- None of the above.

94)Which of the following is the sample space for choosing a letter from the word LIBRARY?

- {I, A}
- {L, I, B, R, A, R, Y}
- {L, I, B, R, A, Y}
- None of the above.

95) What is the probability that a single card chosen from a deck is not an ace?

- 1/13
- 12/13
- 3/4
- None of the above.

96) Which of the following is a certain event?

- Choosing a teacher from a room full of students.
- Choosing an odd number from the numbers 1 to 10.

- Getting a 4 after rolling a single 6-sided die.
- None of the above.

97) There are 4 parents, 3 students and 6 teachers in a room. If a person is selected at random, what is the probability that it is a teacher or a student?

- 9/13
- 4/13
- 7/13
- None of the above.

98) In a high school computer class there are 15 juniors and 10 seniors. Four juniors and five seniors are boys. If a student is selected at random, then what is the probability of selecting a junior or a boy?

- 24/25
- 4/5
- 1/5
- None of the above

99) A jar contains 5 red, 3 green, 2 purple and 4 yellow marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a purple and a red marble?

- 5/98
- 1/2
- 3/98
- 2/49

100) Three cards are chosen at random from a deck without replacement. What is the probability of choosing an eight, a seven and a six, in order?

- 6/35.152
- 1/2197
- 8/16.575
- None of the above.

101)In a shipment of 25 DVD Players, 2 are defective. If 2 DVD Players are randomly selected and tested, what is the probability that both are defective if the first one is not replaced after it has been tested?

- 4/625
- 1/300
- 2/625
- None of the above.

102) In a school, 48% of the students take a foreign language class and 19% of students take both foreign language and technology. What is the probability that a student takes technology given that the students takes foreign language? (Round your answer to the nearest percent.)

- 67%
- 253%
- 40%
- None of the above.

(1) Find the expectation of the sum of points in tossing a pair of fair dice.

(2) Find the expectation of a discrete random variable X whose probability function is given by $f(x) = \left(\frac{1}{2}\right)^x, \; x = 1, 2, 3, \ldots$

(3) A continuous random variable X has probability density given by

$$f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find $E(X), E(X^2)$

(4) The joint density function of two random variables X and Y is given by

$$f(x, y) = \begin{cases} xy/96 & 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

Find $E(X), E(Y), E(XY), E(2X + 3Y)$

(5) Find (a) the variance and (b) the standard deviation of the sum obtained in tossing a pair of fair dice.

(6) Find (a) the variance and (b) the standard deviation for a continuous random variable X which has probability density given by $f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$

(7) Prove that the quantity $E\left[(X - a)^2\right]$ is minimum when $a = \mu = E(X)$.

(8) Prove that $E\left[(X - \mu)^2\right] = E(X) - [E(X)]^2$.

(9) If $X^* = (X - \mu)/\sigma$ is standardized random variable,

prove that (a) $E(X^*) = 0$ (b) $Var(X^*) = 1$

(10) Prove that $Var(X+Y)=Var(X)+Var(Y)$.

(11) If X and Y are two continuous random variables having joint density function f(x, y) show that

$\sigma_{XY}=E(XY)-E(X).E(Y)$.

(12) The joint probability function of two discrete random variables X and Y is given by $f(x,y)=c(2x+y)$ where x and y can assume all integers such that $0\le x\le 2,\ 0\le y\le 3$ and $f(x,y)=0$ otherwise. considering c=1/42 Find,

$(a) E(X),\ (b) E(Y),\ (c)\ E(XY),\ (d)\ E(X^*),$

$(e)\ E(Y^2)$, (f)$Var(X),\ (g)\ Var(Y),\ (h)\ Cov\ (X,\ Y),\ (i)\ \rho$

(13) The joint probability function of two discrete random variables X and Y is given by

$$f(x,y)=\begin{cases} c(2x+y) & 2\le x\le 6,\ 0\le y\le 5 \\ 0 & \text{otherwise} \end{cases}$$

considering c=1/210 Find,

$(a) E(X),\ (b) E(Y),\ (c)\ E(XY),\ (d)\ E(X^*),$

$(e)\ E(Y^2)$, (f)$Var(X),\ (g)\ Var(Y),\ (h)\ Cov\ (X,\ Y),\ (i)\ \rho$

(14) Find the coefficient of (a) skewness and (b) kurtosis of the distribution defined by the normal curve, having density

$$f(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2} \qquad -\infty<x<\infty$$

Exercises (5)

(1) A population consists of the four numbers 3, 7, 11, 15. Consider all possible samples of size two which can be drawn with replacement from this population. Find

(a) The population mean,

(b) The population standard deviation,

(c) The mean of the sampling1 distribution of means,

(d) The stand- standard deviation of the sampling distribution of means. Verify (c) and (d) directly from (a) and (b) by use of suitable formulas. **Ans. (a) 9.0 (b) 4.47 (c) 9.0 (d) 3.16**

(2) Solve Problem (1) if sampling is without replacement.

Ans. (a) 9.0 (b) 4.47 (c) 9.0 (d) 2.58

(3) The weights of 1500 ball bearings are normally distributed with a mean of 22.40 ounces and a standard deviation of 0.048 ounces. If 300 random samples of size 36 are drawn from this population, determine the expected mean and standard deviation of the sampling distribution of means if sampling is done (a) with replacement, (b) without replacement.

Ans. (a) $\mu_{\bar{X}} = 22.40$ oz, $\sigma_{\bar{X}} = 0.008$ oz (b) $\mu_{\bar{X}} = 22.40$ oz, $\sigma_{\bar{X}}$ is slightly less than 0.008 oz

(4) Solve Problem (3) if the population consists of 72 ball bearings.

(a) $\mu_{\bar{X}} = 22.40$ oz, $\sigma_{\bar{X}} = 0.008$ oz (b) $\mu_{\bar{X}} = 22.40$ oz, $\sigma_{\bar{X}} = 0.0057$ oz

(5) In Problem (3), how many of the random samples would have their means

(a) Between 22.39 and 22.41 oz,

(b) Greater than 22.42 oz,

(c) Less than 22.37 oz,

(d) Less than 22.38 or more than 22.41 oz?

Ans. (a) 237 (b) 2 (c) none (d) 24

(6) Certain tubes manufactured by a company have a mean lifetime of 800 hr and a standard deviation of 60 hr. Find the probability that a random sample of 16 tubes taken from the group will have a mean lifetime of

(a) Between 790 and 810 hr,

(b) Less than 785 hr,

(c) More than 820 hr,

(d) Between 770 and 830 hr. **Ans. (a) 0.4972 (b) 0.1587 (c) 0.0918 (d) 0.9544**

(7) Work Problem (6) if a random sample of 64 tubes is taken. Explain the difference. **Ans. (a) 0.8164 (b) 0.0228 (c) 0.0038 (d) 1.0000**

(8) Find the probability that of the next 200 children born

(a) Less than 40% will be boys,

(b) Between 43% and 57% will be girls,

(c) More than 54% will be boys. Assume equal probabilities for births of boys and girls.

Ans. (a) 0.0029 (b) 0.9696 (c) 0.1446

(9) Out of 1000 samples of 200 children each, in how many would you expect to find that

(a) Less than 40% are boys,

(b) Between 40% and 60%) are girls,

(c) 53% or more are girls? **Ans. (a) 2 (b) 996 (c) 218**

(10) Work Problem (8) if 100 instead of 200 children are considered and explain the differences in results. **Ans. (a) 0.0179 (b) 0.8664 (c) 0.1841**

(11) An urn contains 80 marbles of which 60% are red and 40% are white. Out of 50 samples of 20 marbles each selected with replacement from the urn, how many samples can be expected to consist of

(a) Equal numbers of red and white marbles,

(b) 12 red and 8 white marbles,

(c) 8 red and 12 white marbles,

(d) 10 or more white marbles? **Ans. (a) 6 (b) 9 (c) 2 (d) 12**

(12) A manufacturer sends out 1000 lots, each consisting of 100 electric bulbs. If 5% of the bulbs are normally defective, in how many of the lots should we expect

(a) Fewer than 90 good bulbs,

(b) 98 or more good bulbs? **Ans.(a) 19 (b) 126**

(13) A and B manufacture two types of cables, having mean breaking strengths of 4000 and 4500 pounds and standard deviations of 300 and 200 pounds respectively. If 100 cables of brand A and 50 cables of brand B are tested, what is the probability that the mean breaking strength of B will be

(a) At least 600 pounds more than A,

(b) At least 450 pounds more than A1 **Ans. (a) 0.0077 (b) 0.8869**

(14) What are the probabilities in Problem (13) if 100 cables of both brands are tested? Account for the differences. **Ans. (a) 0.0028 (b) 0.9172**

(15) The mean score of students on an aptitude test is 72 points with a standard deviation of 8 points. What is the probability that two groups of students, consisting of 28 and 36 students respectively, will differ in their mean scores by

(a) 3 or more points,

(b) 6 or more points,

(c) Between 2 and 5 points? **Ans(a) 0.2150 (b) 0.0064 (c) 0.4504**

(16) An urn holds 60 red marbles and 40 white marbles. Two sets of 30 marbles each are drawn .with replacement from the urn and their colors are noted. What is the probability that the two sets differ by 8 or more red marbles? **Ans. ٠.٠٤٨٢**

(17) Solve Problem (16) if sampling is without replacement in obtaining each set. **Ans. 0.0188**

(18) Ball bearings of a given brand weigh 0.50 ounces with a standard deviation of 0.02ounces. What is the probability that two lots, of 1000 ball bearings each, will differ in weight by more than 2 ounces? **Ans. 0.0258**

(19) A certain type of electric light bulb has a mean lifetime of 1500 hours and a standard deviation of 150 hours. Three bulbs are connected so that when one burns out, another will go on. Assuming the lifetimes are normally distributed, what is the probability that lighting will take place for

(a) At least 5000 hours,

(b) At most 4200 hours? **Ans. 0.0274 (b)0.01251**

(20) It is found that the lifetimes of television tubes manufactured by a company have a mean of 2000 hours and a standard deviation of 60 hours. If 10 tubes are selected at random find the probability that the sample standard deviation will

(a) Not exceed 60 hours,

(b) Lie between 50 and 70 hours. **Ans. (a) 0.36 (b) 0.49**

Lifetime (hours)	Number of Tubes
300 - 399	14
400 - 499	46
500 - 599	58

600 - 699	76
700 - 799	68
800 - 899	62
900 - 999	48
1000-1099	22
1100-1199	6
Total	400

Table (7)

(21) Table (7) shows a frequency distribution of the lifetimes of 400 radio tubes tested at the L&M Tube Company. With reference to this table determine the

(a) Upper limit of the fifth class,

(b) Lower limit of the eighth class,

(c) Class mark of the seventh class,

(d) Class boundaries of the last class,

(e) Class interval size,

(f) Frequency of the fourth class,

(g) Relative frequency of the sixth class,

(h) Percentage of tubes whose lifetimes do not exceed 600 hours,

(i) Percentage of tubes with lifetimes greater than or equal to 900 hours,

(j) Percentage of tubes whose lifetimes are at least 500 but less than 1000 hrs.

(k) Construct (a) a histogram and (ft) a frequency polygon corresponding to the frequency distribution.

Ans. (a) 799 (b) 1000 (c) 949.5 (d) 1099.5, 1199.5 (e)100(hours) (f) 76
(g) 62/400 = 0.155 or 15.5% (h) 29.5% (i) 19.0% (j) 78.0%

(22) For the data of Problem (21) construct

(a) A relative or percentage frequency distribution,

(b) A relative frequency histogram,

(c) A relative frequency polygon.

(23) Estimate the percentage of tubes of Problem (21) with lifetimes of (a) less than 560 hours, (b) 970 or more hours, (c) between 620 and 890 hours. **Ans. (a) 24% (b) 11% (c) 46%**

(24) The inner diameters of washers produced by a company can be measured to the nearest thou- thousandth of an inch. If the class marks of a frequency distribution of these diameters are given in inches by 0.321, 0.324, 0.327, 0.330, 0.333 and 0.336, find

(a) The class interval size, **Ans. (a) 0.003 in.**

(b) The class boundaries, **Ans. (b) 0.3195, 0.322Б, 0.3255, ..., 0.3375 in.**

(c) The class limits. **Ans.(c) 0.320-0.322, 0.323-0.325, 0.326-0.328, ..., 0.335-0.337**

(25) Table (8) shows the diameters in inches of a sample of 60 ball bearings manufactured by a company. Construct a frequency distribution of the diameters using appropriate class intervals.

0.738	**0.729**	**0.743**	**0.740**	**0.736**	**0.741**	**0.735**	**0.731**	**0.726**	**0.737**
0.728	**0.737**	**0.736**	**0.735**	**0.724**	**0.733**	**0.742**	**0.736**	**0.739**	**0.735**
0.745	**0.736**	**0.742**	**0.740**	**0.728**	**0.738**	**0.725**	**0.733**	**0.734**	**0.732**
0.733	**0.730**	**0.732**	**0.730**	**0.739**	**0.734**	**0.738**	**0.739**	**0.727**	**0.735**
0.735	**0.732**	**0.735**	**0.727**	**0.734**	**0.732**	**0.736**	**0.741**	**0.736**	**0.744**
0.732	**0.737**	**0.731**	**0.746**	**0.735**	**0.735**	**0.729**	**0.734**	**0.730**	**0.740**

Table (8)

(26) For the data of Problem (25) construct

(a) a histogram,

(b) a frequency polygon,

(c) a relative frequency distribution,

(d) a relative frequency histogram,

(e) a relative frequency polygon ,

(f) a cumulative frequency distribution,

(g) a percentage cumulative distribution,

(h) an ogive,

(i) a percentage ogive.

(27) From the results in Problem (26) determine the percentage of ball bearings having diameters

(a) exceeding 0.732 inches,

(b) not more than 0.736 inches,

(c) between 0.730 and 0.738 inches. Compare your results with those obtained directly from the raw data of Table (8).

(28) Work Problem (26) for the data of Problem (21)

(29) A student received grades of 85, 76, 93, 82 and 96 in five subjects. Determine the arithmetic mean of the grades. **Ans. 86**

(30) The reaction times of an individual to certain stimuli were measured by a psychologist to be 0.53, 0.46, 0.50, 0.49, 0.52, 0.53, 0.44 and 0.55 seconds. Determine the mean reaction time of the individual to the stimuli. **Ans.٠.٥٠ s**

(31) A set of numbers consists of six 6's, seven 7's, eight 8's, nine 9's and ten 10's. What is the arithmetic mean of the numbers? **Ans.٨.٢٥**

(32) A student's grades in the laboratory, lecture and recitation parts of a physics course were 71, 78 and 89 respectively,

(a) If the weights accorded these grades are 2, 4 and 5 respectively, what is an appropriate average grade?

(b) What is the average grade if equal weights are used?

Ans. (a) 82 (b) 79

(33) Three teachers of economics reported mean examination grades of 79, 74 and 82 in their classes, which consisted of 32, 25 and 17 students respectively. Determine the mean grade for all the classes.

Ans. 78

(34) The mean annual salary paid to all employees in a company was $5000. The mean annual salaries paid to male and female employees of the company were $5200 and $4200 respectively. Determine the percentages of males and females employed by the company. **Ans. %٢٠ ,%٨٠**

(35) Table (9) shows the distribution of the diameters of the heads of rivets manufactured by a company. Compute the mean diameter. **Ans. ٠.٧٢٦٤٢ inches**

Table (9)

Diameter (inches)	Number of Cables
0.7247 - 0.7249	2
0.7250 - 0.7252	6
0.7253 - 0.7255	8
0.7256 - 0.7258	15
0.7259 - 0.7261	42
0.7262 - 0.7264	68
0.7265 - 0.7267	49
0.7268 - 0.7270	25
0.7271 - 0.7273	18
0.7274 - 0.7276	12
0.7277 - 0.7279	4
0.7280 - 0.7282	1
Total	250

Class	Frequency
10- under 15	3
15- under 20	7
20- under 25	16
25- under 30	12
30- under 35	9

35- under 40	5
40- under 45	2
Total	54
Table (10)	

(36) Compute the mean for the data in Table (10).

Ans. 26.2

(37) Find the standard deviation of the numbers:

(a) 3, 6, 2, 1, 7, 5;

(b) 3.2, 4.6, 2.8, 5.2, 4.4;

(c) 0, 0, 0, 0, 0, 1, 1, 1.

Ans. (a) 2.16 (b) 0.90 (c) 0.484

(38) (a) By adding 5 to each of the numbers in the set 3, 6, 2, 1, 7, 5 we obtain the set 8, 11, 7, 6, 12, 10. Show that the two sets have the same standard deviation but different means. How are the means related? (b) By multiplying each of the numbers 3, 6, 2, 1, 7, 5 by 2 and then adding 5, we obtain the set 11, 17, 9, 7, 19, 15. What is the relationship between the standard deviations and between the means for the two sets? (c) What properties of the mean and standard deviation are illustrated by the particular sets of numbers in (a) and (b)?

(39) Find the standard deviation of the set of numbers in the arithmetic progression 4, 10, 16, 22, ..., 154. **Ans. 45**

(40) (a) Find the standard deviation s of the rivet diameters in Problem (35). (b) What percentage of rivet diameters lie in $(\hat{x} \pm s)$, $(\hat{x} \pm 2s)$, $(\hat{x} \pm 3s)$?

(c) Compare the percentages in

(b) with those which would theoretically be expected if the distribution were normal, and account for any observed differences. **Ans. (a) 0.000576 inches (b) 72.1%, 93.3%, 99.76%**

(41) Find the (a) first, (b) second, (c) third and (d) fourth moment about the origin for the set of numbers 4, 7, 5, 9, 8, 3, 6.

Ans. (a) 6 (b) 40 (c) 288 (d) 2188

(42) Find the (a) first, (b) second, (c) third and (d) fourth moment about the mean for the set of numbers in Problem (43).

Ans. (a) 0 (b) 4 (c) 0 (d) 25.86

(43) Find the (a) first, (b) second, (c) third and (d) fourth moment about the number 7 for the set of numbers in Problem (43). **vAns (a) -1 (b) 5 (c) -91 (d) 53**

(44) Using the results of Problems (43) and (44), verify the following relations between the moments:

$(a)\ m_2 = m'_2 - m'^2_1$

$(b)\ m_3 = m'_3 - 3m'_1 m'_2 + 2m'^3_1$

$(c)\ m_4 = m'_4 - 4m'_1 m'_3 + +6m'^2_1 m'_2 - 3m'^4_1$

(45) Find the first four moments about the mean of the set of numbers in the arithmetic progression 2, 5, 8, 11, 14, 17. **Ans. 0, 26.26, 0, 1193.1**

(46) If the first moment about the number 2 is equal to 5, what is the mean? **Ans.7**

(47) If the first four moments of a set of numbers about the number 3 are equal to -2, 10, -25 and 50, determine the corresponding moments

(a) about the mean,

(b) about the number 5,

(c) about zero. **Ans. (a) 0, 6, 19, 42 (b) -4, 22, -117, 660 (c) 1, 7, 38, 155**

Find the first four moments about the mean of the numbers 0, 0, 0, 1, 1, 1, 1, 1. **Ans. 0, 0.2344, -0.05**

(1) Determine the probability p, or an estimate of it, for each of the following events:

(a) A king, ace, jack of clubs or queen of diamonds appears in drawing a single card from a well shuffled ordinary deck of cards.

(b) The sum 8 appears in a single toss of a pair of fair dice.

(c) A nondefective bolt will be found next if out of 600 bolts already examined, 12 were defective.

(d) A 7 or 11 comes up in a single toss of a pair of fair dice.

(e) At least one head appears in three tosses of a fair coin.

(a) 5/26 (b) 5/36 (c) 0.98, (d) 2/9 (e) 7/8

(2) An experiment consists of drawing three cards in succession from a well-shuffled ordinary deck of cards. Let A1 be the event "king on first draw," A2 the event "king on second draw," and A3 the event "king on third draw." State in words the meaning of each of the following:

$(a)\ P(A_1 \cap A_2'),\qquad (b)\ P(A_1 \cup A_2),\qquad (c)\ P(A_1' \cap A'_2),$

$(d)\ P(A_1' \cap A'_2 \cap A'_3),\qquad (e)\ P\left\{(A_1 \cap \mathrm{A}_2) \cup (\mathrm{A}_2' \cap \mathrm{A}_3)\right\}$

Ans.

(a) Probability of king on first draw and no king on second draw.

(b) Probability of either a king on first draw or a king on second draw or both.

(c) No king on first draw or no king on second draw or both (no king on first and second draws).

(d) No king on first, second and third draws.
(e) Probability of either king on first draw and king on second draw or no king on second draw and king on third draw.

(3) A marble is drawn at random from a box containing 10 red, 30 white, 20 blue and 15 orange marbles. Find the probability that it is

(a) Orange or red,
(b) Not red or blue,
(c) Not blue,
(d) White,
(e) Red, white or blue.

Ans. (a) 1/3 (b) 3/5 (c) 11/15 (d) 2/5 (e) 4/5

(4) Two marbles are drawn in succession from the box of Problem (3), replacement being made after each drawing. Find the probability that (a) Both are white,

(b) The first is red and the second is white,
(c) Neither is orange,
(d) They are either red or white or both (red and white),
(e) The second is not blue,
(f) The first is orange,
(g) At least one is blue,
(h) At most one is red,
(i) The first is white but the second is not,
(j) Only one is red.

Ans. (a)4/25,(b)4/75(c)(16/25),(d)64/225,(e)11/16,(f)1/5,
(g)104/225, (h)221/225,(i)6/25,(j)52/225

(5) Work Problem 4 if there is no replacement after each drawing.

(a)29/185,(b)2/37(c)(118/185),(d)52/185,(e)11/15,(f)1/5,
(g)86/185, (h)182/185,(i)9/37,(j)26/111

(6) A box contains 2 red and 3 blue marbles. Find the probability that if two marbles are drawn at random (without replacement)

(a) Both are blue,
(b) Both are red,
(c) One is red and one is blue.

Ans. (a) 3/10 ,(b) 1/10, (c) 3/5

(7) Find the probability of drawing 3 aces at random from a deck of 52 ordinary cards if the cards are (a) replaced, (b) not replaced.

Ans. (a) 1/2197, (b) 1/17576.

(8) If at least one child in a family with two children is a boy what is the probability that both children are boys? **Ans. 1/3**

(9) If A is independent of B prove that

(a) A is independent of B’,

(b) A' is independent of B'.

(10) If A, B, C are independent events prove that

(a) A and $(B \cup C)$,

(b) A and $(B \cap C)$,

(c) A and $(B - C)$, are independent.

(11) Let A_1 = event "odd number on first die," A_2 = event "odd number on second die," A_3= event "odd total on both dice." Show that $A_1, A_2; A_2, A_3; A_1, A_3$ are independent but that A_1, A_2, A_3 are not independent.

(12) A coin is tossed three times. Use a tree diagram to determine the various possibilities which can arise.

(13) Three cards are drawn at random (without replacement) from an ordinary deck of 52 cards. Use a tree diagram to determine the number of ways in which one can draw (a) a diamond and a club
and a heart in succession, (b) two hearts and then a club or a spade.

(14) In how many ways can 3 different coins be placed in 2 different purses? (8)

(15) In how many ways can 3 men and 3 women be seated at a round table if

(a) No restriction is imposed, 120

(b) Two particular women must not sit together, 72

(c) Each woman is to be between two men? 12

Ans. (a) 120, (b) 72 ,(c) 12

(16) How many different committees of 3 men and 4 women can be formed from 8 men and 6 women? **Ans. 840**

(17) In how many ways can 2 men, 4 women, 3 boys and 3 girls be selected from 6 men, 8 women, 4 boys and 5 girls if

(a) No restrictions are imposed,

(b) A particular man and woman must be selected?

Ans. (a)42000, (b)7000

(18) In how many ways can a group of 10 people be divided into

(a) Two groups consisting of 7 and 3 people,

(b) Three groups consisting of 5, 3 and 2 people?

Ans. (a)120, (b)2520

(19) From 5 statisticians and 6 economists a committee consisting of 3 statisticians and 2 economists is to be formed. How many different committees can be formed if

(a) no restrictions are imposed,

(b) two particular statisticians must be on the committee,

(c) one particular economist cannot be on the committee?

Ans. (a)150, (b)45, (c)100

(20) A box contains 9 tickets numbered from 1 to 9 inclusive. If 3 tickets are drawn from the box one at a time, find the probability that they are alternately either odd, even, odd or even, odd, even. (5/18)

(21) An urn contains 6 red and 8 blue marbles. Five marbles are drawn at random from it without replacement. Find the probability that 3 are red and 2 are blue. ${}^{6}C_3{}^{8}C_2/{}^{14}C_5$

(22) A system fails when a defect occurs in one of 9 subsystems labeled 1, …, 9 (only one defect occurs at a time). Let pi denote the probability that the defect is in subsystem i. Suppose that each of the subsystems 4; 5; 6 is twice as likely to contain the defect as any one of the other subsystems. What is the probability of the defect is in subsystems 1, 2, 3.

(23) A certain product was found to have two types of minor defects. The probability that an item of the product has only a type 1 defect is 0.2, and the probability that an item of the product has only a type 2 defect is 0.3. Also, the probability that it has both defects is 0.1. Find the probabilities of the following events:
A = an item has either a type 1 defect or a type 2 defect.
B = an item does not have either of the defects.
C = an item has defect 1, but not defect 2.
D = an item has exactly one of the two defects.

(24) If a dice is rolled once, what is the probability that it will show a prime number (1 is not prime)?

(25) If a dice is rolled once, what is the probability that it will show a multiple of 1, 3?

(26) If a coin is flipped twice, what is the probability that it will land heads once and tails once?

(27) A bag contains 4 white counters, 6 black counters, and 1 green counter. What is the probability of drawing?

(a) A white counter?
(b) A black counter?
(c) A green counter?
(d) A white counter or a black counter?
(e) A white counter or a green counter?

(28) A fair, six-sided die is rolled. What is the probability of obtaining a 3 or an odd number?

(29) A large basket of fruit contains 3 oranges, 2 apples and 5 bananas. If a piece of fruit is chosen at random, what is the probability of getting an orange or a banana?

(30) Two dice are rolled; find the probability that the sum is

(a) equal to 1

(b) equal to 4

(c) less than 13

(31) Consider the class {A, B, C, D} of events. Suppose the probability that at least one of the events A or C occurs is 0.75 and the probability that at least one of the four events occurs is 0.90.Determine the probability that neither of the events A or C but at least one of the events B or D occurs.

(1)A coin is tossed three times. If X is a random variable giving the number of heads which arise,

(a) Construct a table showing the probability distribution of X and

(b) Graph the distribution.

(c)Obtain the distribution function and its graph.

(2) An urn holds 5 white and 3 black marbles. If two marbles are drawn at random without replacement and X denotes the number of white marbles,

(a) Find the probability distribution for X and

(b) Graph the distribution.

(c) Obtain the distribution function and its graph .

(3) Work Problem 2 if the marbles are drawn with replacement.

(4) Let Z be a random variable giving the number of heads minus the number of tails in two tosses of a fair coin. Find the probability distribution of Z.

(5) Let X be a random variable giving the number of aces in a random draw of 4 cards from an ordinary deck of 52 cards, Construct a table showing the probability distribution of X and graph the distribution. Obtain the distribution function and its graph for

(6) The probability function of a random variable X is shown in given table Construct a table giving the distribution function of X and graph this distribution function.

x	1	2	3
f(x)	1/2	1/3	1/6

Table (8)

(7) Table (9) shows the distribution function of a random variable X.
Determine

(a) the probability function,

(b) $\Pr(1 \le X \le 3),\ \Pr(X \ge 2), \Pr(X < 3), \Pr(X > 1.4)$.

x	1	2	3	4
F(x)	1/8	1/8	1/4	1

Table (9)

(8) A random Variable X has density function

$$f(x,y) = \begin{cases} ce^{-3x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

Find

(a) the constant c,

(b) $\Pr(1 < X < 2),\ \Pr(X \ge 3), \Pr(X < 1)$.

(c) Find the distribution function of the random variable X .

(9) A random Variable X has density function

$$f(x,y) = \begin{cases} cx^2 & 1 \le x \le 2 \\ cx & 2 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find

(d) the constant c,

(e) $\Pr(X > 2),\ \Pr(0.5 < X < 1.5)$.

(f) Find the distribution function of the random variable X .

(10) The joint probability function of two discreate random variables X and Y is given by $f(x,y) = cxy$ for $x = 1,2,3$ and $y = 1,2,3$ and equal zero otherwise. Find

(a) the constant c.

(b) $\Pr(X = 2, Y = 3)$, .

(c) $\Pr(1 < X < 2, Y \le 2)$

(d) $\Pr(X < 2)$

(e) $\Pr(X = 3)$

(f) Find the margent probability function of X and Y .

(g) Determine whether X and Y are independent.

(11) The joint probability function of two discreate random variables X and Y is given by

$$f(x,y) = \begin{cases} c(x^2 + y^2) & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find

(a) the constant c.

(b) $\Pr(X < \frac{1}{2}, Y > \frac{1}{2})$, .

(c) $\Pr(\frac{1}{4} < X < \frac{3}{4})$

(d) $\Pr(Y < \frac{1}{2})$

(e) Find the margent probability function of X and Y .

(f) Determine whether X and Y are independent.

(12) Suppose that the random variables X and Y have joint density function given by

$$f(x,y) = \begin{cases} c(2x+y) & 2 < x < 6,\ 0 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

Find

(a) The constant c,

(b) The marginal distribution functions for X and Y,

(c) The marginal density functions for X and Y,

(d) $\Pr(0 < X < 4, Y > 2)$,

(e) $\Pr(X > 8)$,

(f) $\Pr(X + Y > 4)$,

(g) The joint distribution function,

(h) Whether X and Y are independent.

(13) Let X have the density function

$$f(x) = \begin{cases} 6x(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find a function $Y = h(X)$ which has the density function

$$g(y) = \begin{cases} 12y^3(1-y^2) & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(14) The probability function of a random variable X is given by

$$f(x) = \begin{cases} x^2/81 & -3 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability density for the random variable

$$U = \frac{1}{78}(12 - X).$$

(15) Find the density function of $U = XY$ if the joint density function of X and Y is $f(x,y)$.

(16) If the random variables X and Y have joint density function

$$f(x,y)=\begin{cases} xy/96 & 0<x<4,\ 1<y<5 \\ 0 & \text{otherwise} \end{cases}$$

see exam (10) find

(a) The joint density function of $U = X + 2Y$.

(b) The joint density function of $U = XY^2$

(c) The joint density function of $U = X^2Y$

(17) Let X and Y be random variables having joint density function $f(x,y)$. Prove that the density function of $U = X + Y$ is

$$g(u)=\int_{-\infty}^{\infty} f(v,u-v)\,dv$$

18) The time in minutes between individuals joining the line at an Ottawa Post Office is a random variable with the exponential distribution

$$f(x) = 2e^{-2x},\ (x \geq 0).$$

Find the mean and median time between individuals joining the line and interpret the answers.

19) Verify the formula for the mean of a uniform distribution by computing the integral.

20) Verify the formula for the variance of a uniform distribution by computing the integral.

21) Verify the formula for the variance of an exponential distribution by computing the integral.

22) Show that if X is a random variable with density function $f(x) = ae^{-ax}$ on $[0, +\infty)$, then X has median $[\ln 2/a]$.

23) Show that if X is uniform random variable taking values in the interval [a, b], then X has median (a+b)/2.

24) Find the medians of the random variables with the probability density functions of:

$f(x) = (3/2)(1 - x^2)$ on $[0, 1]$

$f(x) = (3/4)(1 - x^2)$ on $[-1, 1]$

$f(x) = 2x e^{-x^2}$ on $[0, +\infty)$

$f(x) = 2x e^{-x^2}$ on $(-\infty, 0]$

$f(x) = 3/(\pi(1 - x^2)^{1/2})$ on $[1/2, 1]$

25)